Elasticity Optimism

By Jean Imbs and Isabelle Mejean*

On average, estimates of trade elasticities are smaller in aggregate data than at sector level. This is an artefact of aggregation: Estimations performed on aggregate data constrain sector elasticities to homogeneity, which creates a heterogeneity bias. The paper shows such a bias exists in two prominent approaches used to estimate elasticities, which has meaningful consequences for the calibration of the trade elasticity in one-sector, aggregative models. With elasticities calibrated to aggregate data, macroeconomic models can have predictions at odds with the implications of their multi-sector counterparts. They do not when elasticities are calibrated using a weighted average of sector elasticities.
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On average, estimates of the price elasticity of imports are larger in microeconomic data than in the aggregate. This constitutes a puzzle: Even though they are constructed from microeconomic sources, aggregate data imply an elasticity lower than an average of microeconomic estimates. The discrepancy motivated Orcutt's (1950) observation that macroeconomic trade elasticities "have been widely accepted as supporting the view that a depreciation would be ineffective" on countries' trade balance (page 117), an "elasticity pessimism" in his words. This paper shows the systematic discrepancy between micro and macroeconomic estimates exists because of a heterogeneity bias. When the bias is at work, aggregate data cannot recover the true average elasticity implied by microeconomic behavior.

The intuition is straightforward. With well behaved residuals, a regression of aggregate imports on their aggregate price implies an estimate equal to a weighted average of microeconomic elasticities. But in an estimation with macroeconomic aggregates, good-level heterogeneity is mechanically pushed into the residual, which, as a result, correlates systematically with the regressor. This is a classic heterogeneity bias, in the sense defined for instance by Pesaran and Smith (1995).

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What is the direction of the heterogeneity bias? Suppose inelastic products tend to display large price changes. Then, products with volatile prices tend to be inelastic, i.e., display little movement in quantities. And aggregate price changes tend to be associated with little response in quantities, because price changes occur particularly often in inelastic activities. Orcutt (1950) already recognized the damage such systematic correlation could inflict on estimates of trade elasticity based on aggregate data. On page 125, he wrote: "most of the price changes in the historical price indices of imports lumped together were due to price changes of commodities with inelastic demands. Since these price changes were associated with only small quantity adjustments, the estimated price elasticity of all imports might well be low".

Why would price changes be systematically large in inelastic products? The literature offers at least two explanations. First, under imperfect competition, firms that face highly elastic demand choose to limit the price consequences of cost shocks, letting markups vary instead.[1] Thus, prices are stable in elastic activities. Second, tariffs create distortions, that are largest in activities with elastic demand. This provides an incentive for policymakers to choose high tariffs in inelastic activities, and thus create large price differences there. Of course, whether such correlations exist is ultimately an empirical question. This paper verifies they do in U.S data.[2]

The exposition so far has assumed away issues of endogeneity in import demand equations. In practice, prices obviously depend on imported quantities. Such identification problems are the focus of most of the empirical literature on trade elasticities. The paper picks two identification strategies from the literature. The first one builds on an estimation introduced by Feenstra (1994), which supplements a conventional demand equation with a simple supply relation to account for the endogeneity of prices. This method interprets the elasticity as a demand (Armington) parameter. The paper derives the aggregate equivalent to the sectoral approach initially proposed by Feenstra (1994), and establishes the theoretical existence of a heterogeneity bias. The bias depends on the cross-sectional covariance between sectoral elasticities and the variance of prices.

The second identification scheme generalizes the interpretation of trade elasticities within a conventional gravity framework. There, trade elasticities map either with the Armington elasticity, or with the distribution of productivity across firms, as in the Ricardian model due to Eaton and Kortum (2002). The estimation, introduced by Caliendo and Parro (2012), is adapted to aggregate data. Price differences are instrumented with tariffs, a measure of trade costs that is standard in the gravity literature. Again, a heterogeneity bias exists within this

[1]For instance, Ravn, Schmitt-Grohe and Uribe (2010) describe a model where this happens because of habits in consumption.

[2]The bias is further amplified if imports are specialized in inelastic products, since then products that are imported in large quantities have low trade elasticity. It is difficult to think of this property as universal across countries. In practice, it will be shown to explain virtually no part of the difference between elasticity estimates in US data.

estimation strategy, and it depends on the covariance between sectoral elasticities and the dispersion of tariffs across countries.

In U.S data, the gravity approach implies a value for the aggregate elasticity, $\hat{\varepsilon}$, equal to -1.79, while its average across sectors, ε, is equal to -5.64. The correlation between sector elasticity estimates and tariffs is -0.31. The second approach implies an aggregate elasticity estimate of -2, and an average sector value of -4.17. The correlation between sector elasticity estimates and the (relevant) dispersion in prices is -0.13. The differences in elasticity estimates are significant at usual confidence levels. The values obtained are commensurate with conventional estimates obtained from aggregate and (on average) from sectoral data. They illustrate the importance of aggregation in explaining the discrepancy: In the same dataset, and using the same estimator, aggregated data imply values around -1.75, whereas sector data imply average values around -5.[3]

Does the value of the elasticity matter? The paper discusses several applications in various areas of international macroeconomics. Whether the trade elasticity is -1.75 or -5 makes a difference. It is easy to see why. If a parameter value corresponds to a world without microeconomic heterogeneity, then the same must be true of a model calibrated using that value. Of course, a one-sector model is a simplification of a multi-sector world. But the simplification ceases to be warranted if one- and multi-sector versions have fundamentally different predictions. This possibility is illustrated in two models. First, in the conventional international real business cycle framework due to Backus, Kehoe and Kydland (1994). The multi-sector version of the model, calibrated with microeconomic elasticity estimates, has predictions at odds with the one-sector version calibrated with $\hat{\varepsilon}$. But it has the same predictions as the one-sector version calibrated with ε. The same is shown to be true in the recent model due to Arkolakis, Costinot and Rodriguez-Clare (2012).

Two prominent alternative approaches exist that explain the discrepancy between aggregate and microeconomic estimates of the trade elasticity, the "international elasticity puzzle" in the words of Ruhl (2008). Feenstra et al. (2014) introduce a generalization of CES preferences, where the elasticity of substitution between foreign varieties can be different from the Armington elasticity between foreign and domestic goods. Only the latter maps into the price elasticity of imports, and these authors' estimate is close to conventional values coming from aggregate data. Elasticities of substitution between foreign varieties, in contrast,

[3]The comparison is performed between a single estimate arising from the aggregation of microeconomic data, and an average of microeconomic estimates. This comparison is legitimate, in that it addresses the puzzle. But it conflates two transformations: first, data are aggregated; and second, the trade elasticity is constrained to homogeneity in aggregate data. How much of the puzzle actually comes from either? The paper considers the trade elasticity estimated from *pooled* microeconomic data, and constrained to homogeneity across sectors. Then, the only difference with an average of microeconomic estimates comes from a homogeneity constraint. Aggregate and constrained estimates are virtually identical: So it must be the heterogeneity in trade elasticities across sectors that creates a bias in aggregate data. This comparison also rules out an explanation based on the argument that goods' substitutability should inherently decrease with the level of aggregation, since aggregation is held constant here.

take high values, close to microeconomic estimates. But the preferences in Feenstra et al. (2014) continue to imply a conventional linear import demand. Therefore, if the Armington elasticity is heterogeneous across products, a heterogeneity bias can still be at play in the estimates arising from these generalized preferences. Unfortunately, in Feenstra et al. (2014) the identification of Armington elasticities is done in the cross-section of sectors, so that heterogeneity is difficult to establish with precision.[4] When estimating trade elasticities using Feenstra (1994), this paper considers preferences that are simpler, but where Armington elasticities can readily be estimated sector by sector. Because of this tradeoff, the two approaches are complementary.

Ruhl (2008) argues trade estimates of the Armington elasticity are typically obtained in cross-section, whereas in macroeconomics the elasticity is identified in time series. Inasmuch as the former approach focuses on the long-run, it incorporates exporters' low frequency decisions at the extensive margin. Elasticities estimated in the cross-section are therefore higher than in time series. It is possible that the dimension used in estimation, and therefore the horizon at which elasticities are estimated, should matter. But the argument cannot explain the key finding in this paper, that different levels of aggregation are sufficient to explain the elasticity puzzle, using the same estimator, the same time horizon, and the same dimension of the data.

The paper is structured as follows. The two approaches used to estimate trade elasticities are introduced in the next section. The properties of the heterogeneity bias are discussed. Section 3 presents the results. Section 4 discusses the economic relevance of the heterogeneity bias. Section 5 concludes.

I. Estimating Trade Elasticities

This section opens with a general discussion of the heterogeneity bias in trade elasticity estimates, before describing the two identification strategies used to estimate trade elasticities, based on Caliendo and Parro (2012), and on Feenstra (1994). Each sub-section first introduces the estimation at product-level, then discusses the properties of a pooled panel estimation still at product-level but constrained to homogeneity, and finally discusses the estimation performed on aggregate data.

[4]For instance, the estimates are not significantly heterogeneous in their Table 4, and the confidence intervals are wide. This is important, for Feenstra et al. (2014) need the assumption that Armington elasticities are homogeneous to quantify the effects of a devaluation. In this paper, we quantify the effects of a devaluation assuming that the Armington elasticity is not significantly different from the cross-country elasticity. Feenstra et al. (2014) cannot reject this hypothesis in half of the sectors they consider.

A. Heterogeneity Bias

Suppose the true relation between imports of good k and their price is given by:

$$d\ln M^k = c^k + \varepsilon^k \; d\ln P^k + e^k \tag{1}$$

where $d\ln M^k$ is the growth rate of good k imports, $d\ln P^k$ is the change in import prices, c^k is a constant, and e^k is an error term. To focus on the heterogeneity bias, assume the error term is well behaved, so that unbiased estimates of the price elasticity of imports ε^k can be obtained at microeconomic level. This assumption is relaxed in the rest of the paper, starting in the next sub-sections dedicated to identification.

Trade elasticities differ by an amount o^k that averages to zero, so that

$$\varepsilon^k = \varepsilon - o^k$$

Since trade elasticities are negative, highly elastic products correspond to large positive values of o^k. By definition, ε represents both an average of product-level elasticities, and the common component of ε^k across sectors, which aggregate data should identify. In the absence of a heterogeneity bias, this should be the value implied both by aggregate data, and, on average, by microeconomic estimates.

Consider a version of equation (1) that is aggregated across products up to country level:

$$\sum_k m^k \; d\ln M^k = \sum_k m^k c^k + \sum_k m^k \varepsilon^k \; d\ln P^k + \sum_k m^k e^k,$$

This implies

$$d\ln M = c + \varepsilon \; d\ln P + u \tag{2}$$

where $m^k = M^k/M$ measures the share of product k in aggregate imports M, $d\ln M = \sum_k m^k \; d\ln M^k$, $d\ln P = \sum_k m^k \; d\ln P^k$, $c = \sum_k m^k c^k$, and $u = \sum_k m^k e^k - \sum_k m^k o^k \; d\ln P^k$.[5] Equation (2) represents a conventional estimation of the trade elasticity performed on aggregate data: With well-behaved residuals, it should pinpoint the mean elasticity $\varepsilon = \sum_k m^k \varepsilon^k$. But as we now show, in the presence of heterogeneity the residuals in equation (2) can be systematically correlated with the regressor, even if $\sum_k m^k e^k$ is orthogonal to aggregate price changes, i.e. even in the absence of any endogeneity bias.

Let $\hat{\varepsilon}$ denote the point estimate of the trade elasticity obtained from aggregate

[5] In official statistics, the log growth rate of aggregate import prices is computed as $d\ln P_t = \sum_k m^k \; d\ln P_t^k$. How m^k is measured determines the type of price index (Laspeyres, Paasche, Tornqvist or Sato-Vartia).

data. By definition

$$\hat{\varepsilon} = \varepsilon + \frac{cov(d \ln P, u)}{var(d \ln P)}$$

where $cov(.)$ $[var(.)]$ denotes the covariance (variance) operator. A heterogeneity bias exists for non zero values of the covariance between $d \ln P$ and u. Without a endogeneity bias, the covariance can be rewritten:

$$cov(d \ln P, u) = -cov \left(\sum_k m^k \; d \ln P^k, \sum_k m^k o^k \; d \ln P^k \right)$$

In the absence of heterogeneity, $o^k = 0$ for all k, and there is no bias. But as soon as trade elasticities vary systematically across products, $cov(d \ln P, u)$ can take non zero values, and aggregate data yield biased estimates of ε.[6]

B. *Caliendo and Parro (2012)*

The first identification builds on a conventional gravity approach, a linear estimation analogous to equations (1) and (2). The gravity estimation is convenient and flexible, in that the resulting estimate can be interpreted in a wide range of models, with perfect or imperfect competition, and with representative or heterogeneous firms.

SECTOR ESTIMATES. — The gravity estimation considers the cross-section of bilateral trade between pairs of countries i, j. For a given product k, it can be written:

$$\ln s_{ij}^k = \Phi_i^k + \Theta_j^k + a^k D_{ij}^k + \varepsilon^k \; \ln \tau_{ij}^k + e_{ij}^k \tag{3}$$

where the set of regressors includes exporter and importer-specific intercepts, Φ_i^k and Θ_j^k, and symmetric bilateral variables (such as distance) denoted by D_{ij}^k, that are all allowed to vary by product. The dependent variable s_{ij}^k is the market share of country j's imports of product k produced in country i. Since there are importer-specific intercepts, this is equivalent to the value of these imports. τ_{ij}^k is the tariff rate imposed on the value of good k exported from i to j. ε^k is the trade elasticity.

The workings of this equation are well known, and so is the scope of its theoretical relevance. As surveyed by Head and Mayer (2014), estimates of the trade elasticity can be interpreted in a wide variety of models, with perfect or imperfect competition, heterogeneous or representative firms, and perfect or imperfect

[6]Heterogeneity is across products, and time-invariant, but the implied covariance between $d \ln P$ and u is computed in the same dimension used to estimate equation (2), which can be over time, across countries, or both. The corresponding indexes are omitted for clarity.

substitutability of goods' varieties. For instance, Caliendo and Parro (2012) interpret ε^k in a Ricardian model following Eaton and Kortum (2002), where the price elasticity of trade depends on the dispersion of productivity across firms. High dispersion means the identity of the most productive producer is unlikely to change in response to an exogenous shock to relative prices, and the response of imports remains muted. But such an interpretation is not exclusive of more conventional ones, based for instance on the imperfect substitutability of goods across locations, and the reallocation of demand in response to changes in relative prices.

Caliendo and Parro (2012) identify the trade elasticity ε^k thanks to asymmetries in bilateral tariffs. The approach is akin to the method of Tetrads proposed by Head, Mayer and Ries (2010). Each market share s^k_{ij} is first normalized by its reciprocal s^k_{ji}, which eliminates any symmetric bilateral regressors in equation (3). Then these ratios are computed for three distinct pairs of countries, (i,j), (j,h) and (h,i), and combined so that all regressors that are not bilateral and asymmetric drop out. Starting from the gravity equation (3), the estimated equation simplifies into

$$\ln \frac{s^k_{ij}s^k_{jh}s^k_{hi}}{s^k_{ji}s^k_{ih}s^k_{hj}} = \varepsilon^k \ln \frac{\tau^k_{ij}\tau^k_{jh}\tau^k_{hi}}{\tau^k_{ji}\tau^k_{ih}\tau^k_{hj}} + e^k_{ij} + e^k_{jh} + e^k_{hi} - e^k_{ji} - e^k_{ih} - e^k_{hj}$$

where the only remaining variables are bilateral and asymmetric. Let ℓ denote a country triplet ($\ell \equiv (i,j,h)$) and define $\tilde{s}^k_\ell \equiv \frac{s^k_{ij}s^k_{jh}s^k_{hi}}{s^k_{ji}s^k_{ih}s^k_{hj}}$ and $\tilde{\tau}^k_\ell \equiv \frac{\tau^k_{ij}\tau^k_{jh}\tau^k_{hi}}{\tau^k_{ji}\tau^k_{ih}\tau^k_{hj}}$. The gravity equation rewrites

$$\ln \tilde{s}^k_\ell = \varepsilon^k \ln \tilde{\tau}^k_\ell + e^k_\ell \tag{4}$$

where $e^k_\ell \equiv e^k_{ij} + e^k_{jh} + e^k_{hi} - e^k_{ji} - e^k_{ih} - e^k_{hj}$ is a residual. Identification obtains if the unobserved asymmetric trade costs between i and j, e^k_{ij}, are orthogonal to tariffs for each k. Then all that is needed to estimate the trade elasticity ε^k are good-specific data on bilateral trade flows and tariffs.

Pooled Microeconomic Estimates. — Consider an estimation performed on a panel of sector-level data, where trade elasticity estimates are constrained to homogeneity across sectors. A constrained version of equation (4) implies:

$$\ln \tilde{s}^k_\ell = \varepsilon \ln \tilde{\tau}^k_\ell + u^k_\ell \tag{5}$$

Since the true model is given by equation (4) and since $\varepsilon^k = \varepsilon - o^k$, the residual is given by

$$u^k_\ell = e^k_\ell - o^k \ln \tilde{\tau}^k_\ell$$

The first term, e_ℓ^k, is the residual of the sector-level equation, assumed orthogonal to tariffs. In the presence of heterogeneity, the second term is unavoidably correlated with the regressor. It constitutes a heterogeneity bias.

The heterogeneity bias can be characterized by computing the covariance between regressor and residuals, $cov_{\ell k}(\ln \tilde{\tau}_\ell^k, u_\ell^k)$. Throughout the paper, $cov_x(.)$ denotes the covariance operator, computed along dimension x. Here it is computed across country triplets and sectors. Using the definition of the residual, the covariance simplifies to:

$$cov_{\ell k}(\ln \tilde{\tau}_\ell^k, u_\ell^k) = -\frac{1}{K}\sum_k o^k var_\ell(\ln \tilde{\tau}_\ell^k) - cov_k\left[E_\ell(o^k \ln \tilde{\tau}_\ell^k), E_\ell(\ln \tilde{\tau}_\ell^k)\right]$$

where $var_x(.)$ $[E_x(.)]$ denotes the variance (expectation) operator computed in dimension x. The details are left for Appendix A. The first term takes high positive values when the dispersion of tariffs across country triplets is low in elastic sectors (i.e. those with high o^k). When that happens, pooled microeconomic data imply a trade elasticity estimate, $\bar{\varepsilon}$, that is systematically closer to zero than its true value ε. Consistent with the intuition in equations (1) and (2), the heterogeneity bias depends on the correlation between the value of the trade elasticity at sector level, and the second moments of prices. Here, these second moments are computed in the dimension that is relevant to the identification of the gravity equation, i.e. in cross section. They are captured by differences in bilateral tariffs.

The second term captures the covariance between an average of tariffs, $E_\ell(\ln \tilde{\tau}_\ell^k)$, and an average of the residual in equation (5), $E_\ell(u_\ell^k)$, where both terms vary across sectors. Its magnitude is an empirical matter.

Why would the cross-country variance of tariffs be systematically lower in elastic sectors? It is an implication of Grossman and Helpman (1994). In their model, tariffs differ across countries for exogenous reasons, e.g. the influence of lobbies. They are also systematically low in activities with elastic demand, where they are most distortionary. Thus, the cross-country variance in tariffs, $var_\ell(\ln \tilde{\tau}_\ell^k)$, is high in sectors with high tariffs, since high tariffs act to magnify the exogenously given cross-country dispersion in tariffs. This simple argument can explain why o^k and $var_\ell(\ln \tilde{\tau}_\ell^k)$ covary negatively.[7]

The main results in this section are summarized in Theorem 1.

ASSUMPTION 1: *For each sector, e_ℓ^k is orthogonal to bilateral tariff ratios computed over country triplets, $\tilde{\tau}_\ell^k$.*

ASSUMPTION 2: *In elastic sectors, tariffs tend to be less dispersed across country triplets.*

[7]The intuition only requires an exogenous source of tariff differences across countries. Then, these differences are magnified in sectors with high tariffs, since tariffs are uniformly high there, across all countries.

THEOREM 1: *Under assumptions 1-2, estimates of the price elasticity of trade obtained with the gravity equation in Caliendo and Parro (2012) are systematically biased towards zero in pooled sectoral data. The bias increases with the absolute value of* $E_k\left(o^k\ var_\ell(\ln\tilde{\tau}_\ell^k)\right)$.

Proof: See Appendix A.

Aggregate Estimates. — Consider now a version of the gravity estimation that is aggregated at country level. By definition, the share of country i in j's total imports is given by:

$$\ln s_{ij} = \ln \frac{\sum_k M_{ij}^k}{\sum_i \sum_k M_{ij}^k} = \ln \sum_k m_j^k s_{ij}^k \simeq \sum_k m_j^k \ln s_{ij}^k$$

As in Section 2.1, M_{ij}^k is the value of good k imports from country i into country j, and $m_j^k \equiv \frac{\sum_i M_{ij}^k}{\sum_i \sum_k M_{ij}^k}$ is country j's total import share in sector k. The third equality constitutes an approximation, that represents the discrepancy between the arithmetic and geometric averages of sector-level market shares.[8]

The sector-specific gravity equation (3) readily aggregates up to country-level:

$$\text{(6)}\quad \ln s_{ij} = \sum_k m_j^k \Phi_i^k + \sum_k m_j^k \Theta_j^k + \sum_k m_j^k a^k D_{ij} + \sum_k m_j^k \varepsilon^k\ \ln\tau_{ij}^k + \sum_k m_j^k e_{ij}^k$$

Following analogous logic with the sector-level estimation to compute Tetrads of country pairs, it is easy to obtain an aggregate equivalent to equation (4):

$$\text{(7)}\qquad \ln\tilde{s}_\ell = \varepsilon \ln\tilde{\tau}_\ell + u_\ell$$

where $\tilde{s}_\ell \equiv \frac{s_{ij}s_{jh}s_{hi}}{s_{ji}s_{ih}s_{hj}}$ is the aggregate counterpart of $\tilde{s}_\ell^k$, and $\tilde{\tau}_\ell \equiv \frac{\tau_{ij}\tau_{jh}\tau_{hi}}{\tau_{ji}\tau_{ih}\tau_{hj}}$ is the aggregate counterpart of $\tilde{\tau}_\ell^k$, with $\ln\tau_{ij} = \sum_k m_j^k\ \ln\tau_{ij}^k$. Crucially, the additive property of the heterogeneity in trade elasticities, $\varepsilon^k = \varepsilon - o^k$, implies a residual given by

$$u_\ell = \sum_k m^k e_\ell^k - \sum_k m^k o^k \ln\tilde{\tau}_\ell^k$$

Equation (7) also requires that import weights be homogeneous across country pairs, for which a sufficient condition is $m_j^k = m^k$. The constraint is assumed for tractability here, but is not imposed in the actual estimations.

Consider the definition of u_ℓ. The first term is an aggregate version of the residual e_ℓ^k in the previous sections. It is orthogonal to $\tilde{\tau}_\ell$ provided unobserved

[8]The difference between $\ln\left[\sum_{k=1}^K m_j^k\ s_{ij}^k\right]$ and $\sum_{k=1}^K m_j^k \ln s_{ij}^k$ is Theil's entropy measure, applied to the distribution of country i's market share across sectors.

asymmetric costs e_{ij}^k are orthogonal to sector-level tariffs τ_{ij}^k both within and between sectors. This condition is more stringent than in the previous section, where the estimation is performed at sector level. Assume this condition holds for now.

The second term is new. It takes non zero values in the presence of heterogeneity in trade elasticities, $o^k \neq 0$ for some k. The corresponding heterogeneity bias can be characterized by computing the covariance between regressor and residuals, $cov_\ell(\ln \tilde{\tau}_\ell, u_\ell)$. It simplifies as follows

$$cov_\ell(\ln \tilde{\tau}_\ell, u_\ell) = -\sum_k {m^k}^2 o^k var_\ell(\ln \tilde{\tau}_\ell^k) - \sum_k \sum_{k' \neq k} m^k m^{k'} o^k cov_\ell(\ln \tilde{\tau}_\ell^k, \ln \tilde{\tau}_\ell^{k'})$$

The details are left for Appendix A. The first term takes high positive values when the dispersion of tariffs across country triplets is low in elastic sectors (i.e. those with high, positive o^k). It is analogous to the heterogeneity bias described in the previous section that plagues pooled estimations. The only difference are the import weights m^k used to aggregate the data. The exact same argument as in the previous section can account for a systematic covariance between the trade elasticity in a given sector and the dispersion in tariff Triads, $var_\ell(\ln \tilde{\tau}_\ell^k)$. If this covariance is negative, then the gravity estimation performed on aggregate data yields a value for the trade elasticity, $\hat{\varepsilon}$, that is systematically closer to zero than ε.

In addition $cov_\ell(\ln \tilde{\tau}_\ell, u_\ell)$ contains a second term: Its value depends on the covariance in sector tariffs across country triplets, and whether this covariance correlates systematically with the trade elasticity at sector level. If elastic sectors tend to have low $cov_\ell(\ln \tilde{\tau}_\ell^k, \ln \tilde{\tau}_\ell^{k'})$, this expression reinforces the bias, with implied estimated values of ε even closer to zero. Its magnitude is an empirical matter.

The main results in this section are summarized in Theorem 2.

ASSUMPTION 3: *e_ℓ^k is orthogonal to bilateral tariff ratios computed over country triplets, $\tilde{\tau}_\ell^k$, both within and between sectors.*

ASSUMPTION 4: *Import weights are identical across countries: $m_j^k = m^k$ for all j.*

THEOREM 2: *Under assumptions 2-3-4, estimates of the price elasticity of trade obtained from aggregate data using the gravity equation in Caliendo and Parro (2012) are systematically biased towards zero. The bias increases with the absolute value of $E_k\left(o^k \ var_\ell(\ln \tilde{\tau}_\ell^k)\right)$. It increases further if inelastic sectors constitute a large share of imports, $E_k\left(m^k \ o^k\right) < 0$.*

Proof: See Appendix A.

There are three reasons why the trade elasticity estimate that arises from aggregate data can differ from that, constrained to homogeneity, implied by microeconomic panel data. Two of them are apparent from Theorems 1 and 2.

First the import weights m_j^k are actually not equal across countries, which Theorem 2 imposes. Second, Assumption 3 is more stringent that Assumption 1, and a conventional endogeneity bias exists in aggregate data (but not in pooled microeconomic data) if tariffs $\tilde{\tau}_\ell^k$ correlate with $e_\ell^{k'}$ but not with e_ℓ^k for any k, k'. Finally, the two additional covariance terms, $-cov_k\left[E_\ell(o^k \ln \tilde{\tau}_\ell^k), E_\ell(\ln \tilde{\tau}_\ell^k)\right]$ and $-\sum_k \sum_{k' \neq k} m^k m^{k'} o^k cov_\ell(\ln \tilde{\tau}_\ell^k, \ln \tilde{\tau}_\ell^{k'})$, are both liable to drive a difference between $\hat{\varepsilon}$ and $\bar{\varepsilon}$.

C. *Feenstra (1994)*

This is a model with imperfect competition, where trade elasticities reflect the willingness of a representative consumer to reallocate demand across imperfectly substitutable varieties in response to changes in relative prices. The trade elasticity maps into the elasticity of substitution between varieties, which Feenstra (1994) estimates. Two assumptions are key: (i) A variety is associated with a country, and (ii) The elasticity of substitution between varieties is constant, a "CES demand system" in the terminology introduced by Arkolakis, Costinot and Rodriguez-Clare (2012).

Microeconomic Estimates. — The workings of a demand system based on Constant Elasticity of Substitution preferences are well known. The model implies the following import demand equation,

$$d \ln s_{it}^k = \varepsilon^k \ d \ln P_{it}^k + \Phi_t^k + \xi_{it}^k \tag{8}$$

where i denotes a variety, i.e. an origin country. s_{it}^k is the market share of country i in expenditures on good k.[9] Since $\varepsilon^k < 0$, the market share decreases with the price of the variety produced in i, P_{it}^k. The intercept Φ_t^k is time-varying and common across countries, and ξ_{it}^k is an error term combining preference shocks and trade costs. The shocks are assumed to be independent and identically distributed across sectors and countries, i.e., $E_t(\xi_{it}^k \xi_{i't}^{k'}) = 0$ for all k, k', i, and i'. Equation (8) constitutes a log-linear import demand function analogous to equation (1).

To account for the endogeneity of prices, Feenstra (1994) imposes a simple supply structure:

$$P_{it}^k = \exp(\upsilon_{it}^k) \left(C_{it}^k\right)^{\frac{\omega^k}{1-\omega^k}}$$

where C_{it}^k is the real consumption of good k imported from country i, and ω^k maps into the price elasticity of supply in sector k. The technology shock υ_{it}^k is independent and identically distributed across sectors and countries, $E_t(\upsilon_{it}^k \upsilon_{i't}^{k'}) =$

[9] The bilateral dimension is not relevant to Feenstra's estimation, and the notation is simplified accordingly. The index i denotes the exporting country. The estimation is presented from the point of view of the importing country.

0 for all k, k', i, i'. After rearranging, this implies

$$d\ln P_{it}^{k} = \omega^{k} d\ln s_{it}^{k} + \Psi_{t}^{k} + \delta_{it}^{k} \tag{9}$$

where Ψ_{t}^{k} is a time-varying intercept common across countries, and $\delta_{it}^{k} = \left(1-\omega^{k}\right) d\upsilon_{it}^{k}$ is an error term that depends on supply shocks. Solve equation (8) for ξ_{it}^{k}, and equation (9) for δ_{it}^{k}, express both in deviations from a reference country r, and multiply term for term to obtain:

$$Y_{it}^{k} = \psi_{1}^{k}\ X_{1it}^{k} + \psi_{2}^{k}\ X_{2it}^{k} + e_{it}^{k} \tag{10}$$

where $Y_{it}^{k} = (d\ln P_{it}^{k} - d\ln P_{rt}^{k})^{2}$, $X_{1it}^{k} = (d\ln s_{it}^{k} - d\ln s_{rt}^{k})^{2}$, $X_{2it}^{k} = (d\ln s_{it}^{k} - d\ln s_{rt}^{k})(d\ln P_{it}^{k} - d\ln P_{rt}^{k})$, and $e_{it}^{k} = -\left(\xi_{it}^{k} - \xi_{rt}^{k}\right)\left(\delta_{it}^{k} - \delta_{rt}^{k}\right)\frac{1}{\varepsilon^{k}}$.

The shocks embedded in e_{it}^{k} can correlate systematically with the regressors: endogeneity is therefore an issue in equation (10). Feenstra (1994) observes that the time average of e_{it}^{k} is zero, provided the shocks ξ_{it}^{k} and δ_{it}^{k} are uncorrelated across countries. The time averages of X_{1it}^{k} and X_{2it}^{k}, denoted $\bar{X}_{1i}^{k}$ and $\bar{X}_{2i}^{k}$, constitute therefore appropriate instruments in equation (10), since $cov_{it}(\bar{X}_{1i}^{k}, e_{it}^{k}) = cov_{it}(\bar{X}_{2i}^{k}, e_{it}^{k}) = 0$. They solve the issue of endogeneity present in the import demand equation.[10] Since they are averages over time, identification is effectively obtained across countries.

Estimates of equation (10) map directly with the parameters of interest, since

$$\psi_{1}^{k} = -\frac{\omega^{k}}{\varepsilon^{k}}, \qquad \psi_{2}^{k} = \omega^{k} + \frac{1}{\varepsilon^{k}}$$

The estimated value for ε^{k} is given by

$$\varepsilon^{k} = \frac{\psi_{2}^{k} + \sqrt{{\psi_{2}^{k}}^{2} + 4\psi_{1}^{k}}}{-2\psi_{1}^{k}}, \quad \psi_{1}^{k} > 0$$

See Appendix B for details, including the computation of standard errors. What makes it possible to estimate the trade elasticity at microeconomic level is that identification (with instruments) is across countries: equation (10) can be estimated separately for each sector.

It will be convenient to solve the system formed by equations (8) and (9) in

[10]In practice, an intercept is included in equation (10) to account for the measurement error arising from the unit values used to approximate prices. Given the origin of potential measurement error, the intercept is allowed to vary at the most disaggregated level, i.e. for each HS6 category.

terms of the two structural shocks. The system implies

$$d\ln s^k_{it} - d\ln s^k_{rt} = z^k_s(\delta^k_{it} - \delta^k_{rt}) + b^k_s(\xi^k_{it} - \xi^k_{rt}) \tag{11}$$
$$d\ln P^k_{it} - d\ln P^k_{rt} = z^k_P(\delta^k_{it} - \delta^k_{rt}) + b^k_P(\xi^k_{it} - \xi^k_{rt}) \tag{12}$$

where z^k_s (z^k_P) denotes the response of market shares (prices) to technology shocks, and b^k_s (b^k_P) denotes the response of market shares (prices) to the preference shock and changes in transport costs that are embedded in ξ. We have:

$$\begin{array}{lll}
z^k_s \equiv \frac{\varepsilon^k}{1-\varepsilon^k\omega^k} < 0, & \frac{dz^k_s}{d\varepsilon^k} > 0 & \frac{dz^k_s}{d\omega^k} > 0 \\
b^k_s = z^k_P \equiv \frac{1}{1-\varepsilon^k\omega^k} > 0, & \frac{db^k_s}{d\varepsilon^k} > 0 & \frac{db^k_s}{d\omega^k} < 0 \\
b^k_P \equiv \frac{\omega^k}{1-\varepsilon^k\omega^k} > 0, & \frac{db^k_P}{d\varepsilon^k} > 0 & \frac{db^k_P}{d\omega^k} > 0
\end{array}$$

Pooled Microeconomic Estimates. — Consider the constraint that trade elasticity estimates are homogeneous across sectors, in a pool of microeconomic data. Imposing this constraint, equation (10) becomes:

$$Y^k_{it} = \psi_1\ X^k_{1it} + \psi_2\ X^k_{2it} + u^k_{it} \tag{13}$$

where $\psi_1 = -\frac{\omega}{\varepsilon}$, and $\psi_2 = \omega + \frac{1}{\varepsilon}$.[11] The residual is given by

$$u^k_{it} = -\frac{1}{\varepsilon}(\xi^k_{it} - \xi^k_{rt})(\delta^k_{it} - \delta^k_{rt}) + \frac{o^k}{\varepsilon}(\delta^k_{it} - \delta^k_{rt})(d\ln P^k_{it} - d\ln P^k_{rt})$$

The first term averages to zero over time, provided $E_t(\xi^k_{it}\ \delta^k_{it}) = 0$ for all k, i.e., provided technology and demand shock are orthogonal in each sector. This is the same requirement as in the previous section. As a consequence, the same IV strategy continues to resolve the endogeneity embedded in the first term of u^k_{it}. The second term, however, does not average to zero over time. This constitutes the heterogeneity bias in a pooled sector-level estimation of ε.[12]

The heterogeneity bias in the instrumented estimation of Equation (13) can be characterized by computing $cov_{ikt}(\bar{X}^k_{1i}, u^k_{it})$ and $cov_{ikt}(\bar{X}^k_{2i}, u^k_{it})$, where $\bar{X}^k_{1i}$ and $\bar{X}^k_{2i}$ denote the time averages of X^k_{1it}, and the covariances are computed in the

[11] Just like for sector-specific estimations of ε^k, an intercept that varies by HS6 category is included for the estimation of equation (13).

[12] The price elasticity of supply is also assumed to be homogeneous. Appendix C generalizes the results to a case with heterogeneous ω^k, and spells out the conditions under which this will create an additional bias.

dimension of the panel.[13] After some algebra, equations (11)-(12) imply:

$$\begin{aligned} cov_{ikt}(\bar{X}_{1i}^k, u_{it}^k) &= \frac{1}{\varepsilon}\frac{1}{K}\sum_k o^k \cdot z_P^k \cdot \left(z_s^k\right)^2 \cdot var_i(\sigma^2_{\delta_i^k}) \\ cov_{ikt}(\bar{X}_{2i}^k, u_{it}^k) &= \frac{1}{\varepsilon}\frac{1}{K}\sum_k o^k \cdot \left(z_P^k\right)^2 \cdot z_s^k \cdot var_i(\sigma^2_{\delta_i^k}) \end{aligned}$$

where $\sigma^2_{\delta_i^k} = var_t(\delta_{it}^k - \delta_{rt}^k)$ is the time variance of technology shocks in sector k and country i, and $var_i(\sigma^2_{\delta_i^k})$ denotes the cross-country variance in $\sigma^2_{\delta_i^k}$. The details are left for Appendix C.

By definition, $\frac{1}{\varepsilon} \cdot \left(z_P^k\right)^2 \cdot z_s^k > 0$, and $\frac{1}{\varepsilon} \cdot z_P^k \cdot \left(z_s^k\right)^2 < 0$. Heterogeneity therefore affects ψ_1 and ψ_2 in opposite directions. If the covariance between o^k and $var_i\left(\sigma^2_{\delta_i^k}\right)$ is negative, the estimation of equation (13) yields estimates of ψ_1 that are biased upwards, and estimates of ψ_2 that are biased downwards. Given the definition of ε, this implies the estimated trade elasticity is biased towards zero.

Why would prices be systematically stable in elastic sectors? It can happen in the presence of habit formation. Then firms seek to preserve market shares, and use markups to offset cost shocks. They minimize changes in prices. This tends to happen everywhere, so that $var_i\left(\sigma^2_{\delta_i^k}\right)$ takes low values.[14] The intuition is once again identical to Section 2.1, with the additional wrinkle that the endogeneity of prices in the linear demand equation is treated with the supply equation (9).

This section's main results are summarized in Theorem 3.

ASSUMPTION 5: *δ_{it}^k and ξ_{it}^k are independent and identically distributed across countries in each sector.*

ASSUMPTION 6: *In elastic sectors, prices tend to be stable in all countries, conditional on supply shocks: $E_k\left(o^k \ var_i(\sigma^2_{\delta_i^k})\right) < 0$.*

THEOREM 3: *Under assumptions 5-6, estimates of the price elasticity of trade obtained in pooled sectoral data with the system in Feenstra (1994) are systematically biased towards zero. The bias increases with the absolute value of $E_k\left(o^k \ var_i(\sigma^2_{\delta_i^k})\right)$.*
Proof: See Appendix C.

[13]Technically, the bias also depends on the covariance between the regressors. For exposition purposes, the rest of the demonstration assumes this covariance to be null. The expression for the heterogeneity bias thus derived is well supported by the subsequent empirical results.

[14]This narrative would require interpreting the structural shock v_{it}^k as a combination of supply shocks and an endogenous markup response.

Aggregate Estimates. — Consider an aggregation of the growth rates in product prices and in market shares to country level. Aggregate price changes for the goods imported from country i are given by:

$$d\ln P_{it} = \sum_{k=1}^{K} m_{it-1}^k \; d\ln P_{it}^k$$

where $m_{it}^k = \frac{P_{it}^k C_{it}^k}{\sum_{k=1}^K P_{it}^k C_{it}^k}$ is the share of sector k in imports from country i at time t.[15] The change in aggregate market shares is in turn approximately equal to

$$d\ln s_{it} \simeq \sum_{k=1}^{K} m_{it-1}^k \; d\ln s_{it}^k$$

The derivation is left for Appendix D. It requires that expenditures shares be constant. This is verified if preferences are Cobb-Douglas across products, which is assumed from now on.

Since both demand and supply are linear, they aggregate readily at country level, which implies the following system:

$$\begin{aligned} d\ln s_{it} &= \sum_k m_i^k \cdot \varepsilon^k \cdot d\ln P_{it}^k + \sum_k m_i^k \Phi_t^k + \sum_k m_i^k \xi_{it}^k \\ d\ln P_{it} &= \sum_k m_i^k \cdot \omega^k \cdot d\ln s_{it}^k + \sum_k m_i^k \Psi_t^k + \sum_k m_i^k \delta_{it}^k \end{aligned}$$

The paper focuses on the aggregate consequences of heterogeneous trade elasticities across sectors. Other potential sources of heterogeneity are assumed away, including import shares. For tractability, they are assumed to be homogeneous across countries, i.e. $m_i^k = m^k$. This does not invalidate the substance of what follows, nor is it consequential empirically.[16]

With these assumptions, it is easy to solve aggregate demand for $\sum_k m^k \xi_{it}^k$, aggregate supply for $\sum_k m^k \delta_{it}^k$, express both in deviations from the benchmark country r, and multiply term for term to obtain

$$Y_{it} = \psi_1 \; X_{1it} + \psi_2 \; X_{2it} + u_{it} \tag{14}$$

where $Y_{it} = (d\ln P_{it} - d\ln P_{rt})^2$, $X_{1it} = (d\ln s_{it} - d\ln s_{rt})^2$, $X_{2it} = (d\ln s_{it} - d\ln s_{rt})(d\ln P_{it} - d\ln P_{rt})$, $\psi_1 = -\frac{\omega}{\varepsilon}$, and $\psi_2 = \omega + \frac{1}{\varepsilon}$.[17] Crucially, the residual is

[15] The definition of the price index depends on how m_{it}^k is measured. Here the price index is Laspeyres because import shares are measured in $t-1$.

[16] The price elasticity of supply ω is also assumed to be homogeneous. Appendix C discusses the additional bias that arises if ω is allowed to be heterogeneous.

[17] In sectoral data, the correction for measurement error involves the inclusion of an intercept that

given by

$$\begin{aligned} u_{it} &= -\frac{1}{\varepsilon}\left(\sum_k m^k(\xi_{it}^k - \xi_{rt}^k)\right)\left(\sum_k m^k(\delta_{it}^k - \delta_{rt}^k)\right) \\ &+ \frac{1}{\varepsilon}\left(\sum_k m^k(\delta_{it}^k - \delta_{rt}^k)\right)\left(\sum_k m^k o^k(d\ln P_{it}^k - d\ln P_{rt}^k)\right) \end{aligned}$$

where o^k still denotes the deviation of sector k's trade elasticity with respect to the average: $o^k \equiv \varepsilon - \varepsilon^k$.

The first term is analogous to the residual e_{it}^k, introduced in the sector-level estimation. Its average over time is equal to zero, provided $E_t(\xi_{it}^k\ \delta_{it}^{k'}) = 0$ for all k, k', i.e., provided technology shocks are orthogonal to demand shocks across countries *and* sectors. Then, time averages of the regressors are valid instruments, as they were at sector level. The requirement is more stringent here than in the sector-level estimation, and it is invalid in the presence of basic input-output linkages. Assume for now $E_t(\xi_{it}^k\ \delta_{it}^{k'}) = 0$ for all k, k'.

The second term constitutes the heterogeneity bias. It can be characterized by computing $cov_{it}(\bar{X}_{1i}, u_{it})$ and $cov_{it}(\bar{X}_{2i}, u_{it})$, where $\bar{X}_{1i}$ ($\bar{X}_{2i}$) denote the time averages of X_{1it} (X_{2it}). After some algebra, equations (11)-(12) imply:

$$\begin{aligned} cov_{it}(\bar{X}_{1i}, u_{it}) &= \frac{1}{\varepsilon}\sum_k \left(m^k\right)^4 \cdot o^k \cdot z_P^k \cdot \left(z_s^k\right)^2 \cdot var_i(\sigma^2_{\delta_i^k}) \\ cov_{it}(\bar{X}_{2i}, u_{it}) &= \frac{1}{\varepsilon}\sum_k \left(m^k\right)^4 \cdot o^k \cdot \left(z_P^k\right)^2 \cdot z_s^k \cdot var_i(\sigma^2_{\delta_i^k}) \end{aligned}$$

The details are left for Appendix C. The expressions are similar to the results obtained in pooled microeconomic data, up to the weights used in aggregating the data. By analogy, a negative covariance between o^k and $var_i(\sigma^2_{\delta_i^k})$ implies that estimates of ψ_1 obtained from equation (14) are biased upwards, and estimates of ψ_2 are biased downwards. This implies the estimated aggregate trade elasticity is biased towards zero, $\hat{\varepsilon} > \varepsilon$, consistent with the elasticity puzzle. The bias is reinforced if, in addition, o^k and m^k covary negatively, i.e. inelastic sectors are open.

The main results in this section are summarized in Theorem 4.

ASSUMPTION 7: *δ_{it}^k and ξ_{it}^k are independent and identically distributed across countries and across sectors.*

ASSUMPTION 8: *Import weights are constant and identical across countries: $m_{it}^k = m^k$ for all i, t.*

varies at the HS6 level. Because measurement error washes out in the aggregate, no such intercept is included in equation (14).

THEOREM 4: *Under assumptions 6-7-8, estimates of the price elasticity of trade obtained in aggregate data with the system in Feenstra (1994) are systematically biased towards zero. The bias increases with the absolute value of* $E_k\left(o^k\ var_i(\sigma^2_{\delta^k_i})\right)$. *It increases further if inelastic sectors constitute a large share of imports.*
Proof: See Appendix C.

In principle, there are two reasons why the trade elasticity estimate that arises from aggregate data can differ from that, constrained to homogeneity, implied by microeconomic panel data. They both come from comparing Theorems 3 and 4. First the import weights m^k_{it} are actually not constant or equal across countries, which is imposed in Theorem 4. Second, Assumption 7 is more stringent that Assumption 5, and a conventional endogeneity bias exists in aggregate data (but not in pooled microeconomic data) if δ^k_{it} correlates with $\xi^{k'}_{it}$, but not with ξ^k_{it}, for any k, k'.

D. The Response of Trade to Macroeconomic Shocks

Sections 2.2 and 2.3 adapt two well-known approaches to estimate trade elasticities at micro level, at aggregate level, and in a panel of micro data where the coefficient is constrained to homogeneity. Both sections document the econometric reasons why, in the latter two cases, a heterogeneity bias plagues trade elasticity estimates. This section introduces the structural shock to prices that triggers a change in trade, building on a general multi-sector model with heterogeneity. The paper asks whether heterogeneity can explain the discrepancy between trade elasticity estimates based on disaggregated vs. macroeconomic data. For the comparison to be meaningful, the fundamental shock must be the same in either case. Since the comparison involves a macroeconomic trade elasticity, i.e., the price elasticity of *aggregate* imports, the experiment must consider a macroeconomic shock, i.e. one that affects all relative prices uniformly. The rest of this section describes the consequences of such a shock.

Focus on the trade elasticity in one specific country of interest, indexed with j, whose bilateral trade patterns with a range of partners $i = 1, ..., I$ are used for identification as described in the previous two sections. Define η_j the price elasticity of aggregate imports there:

$$\eta_j \equiv \frac{d\ln \sum_k \sum_{i\neq j} P^k_{ij}C^k_{ij}}{d\ln p^M_j} = \sum_k \sum_{i\neq j} w^k_{ij} m^k_j \frac{d\ln P^k_{ij}C^k_{ij}}{d\ln p^M_j} \tag{15}$$

Aggregate imports at the numerator sum the value of imports across sectors k and origin countries i. The denominator represents an exogenous shock to the relative price of aggregate imports p^M_j in country j. In Caliendo and Parro (2012), this is identified using Tetrads of bilateral tariffs. In Feenstra (1994), this is identified with supply shocks to relative prices. The second equality introduces the share of imports of goods k produced in country i, $w_{ij}^k \equiv \frac{P_{ij}^k C_{ij}^k}{\sum_{i \neq j} P_{ij}^k C_{ij}^k}$.

Costinot and Rodriguez-Clare (2014) show that, for a broad range of trade models, the import share of country i in imports of good k into country j is given by

$$s_{ij}^k \equiv \frac{P_{ij}^k C_{ij}^k}{\sum_{i \neq j} P_{ij}^k C_{ij}^k} = \left(\frac{P_{ij}^k}{P_j^k} \right)^{1-\sigma^k} \frac{1}{\lambda_j^k} \tag{16}$$

where P_j^k is the price index of good k in country j, inclusive of both domestically produced and imported varieties, $\lambda_j^k \equiv \frac{\sum_{i \neq j} P_{ij}^k C_{ij}^k}{\sum_i P_{ij}^k C_{ij}^k}$ is the share of imports in j's consumption of good k, and σ^k is the elasticity of substitution between varieties of good k. Under conventional calibrations, equation (16) holds across most trade models, with perfect or imperfect competition, Armington-based imperfect substitutability or Ricardian trade, and representative or heterogeneous firms, with or without entry. See Costinot and Rodriguez-Clare (2014) for details. Importantly, either of the approaches used to identify ε^k in this paper are special cases of equation (16).

The price elasticity of imports is implied directly by equation (16). Consider first a *microeconomic* shock to the price of good k imported from country i, say $d \ln {p_{ij}^k}^M$. In general, imports from country i represent a small share of country j's imports of good k, so that $d \ln {p_{ij}^k}^M$ leaves the price index P_j^k unchanged. The price elasticity of imports is then given by $\frac{d \ln P_{ij}^k C_{ij}^k}{d \ln {p_{ij}^k}^M} = (1 - \sigma^k) \frac{d \ln P_{ij}^k}{d \ln {p_{ij}^k}^M}$, which is precisely ε^k, the object defined in Section 2.1, and identified by either of the methods just described.[18]

But if instead all import prices are affected by a *macroeconomic* shock, say $d \ln {p_j}^M$, across all sectors and all exporters i, then the price elasticity of imports is equal to

$$\frac{d \ln P_{ij}^k C_{ij}^k}{d \ln {p_j}^M} = \varepsilon^k - (1 - \sigma^k) \frac{d \ln P_j^k}{d \ln {p_j}^M} = \varepsilon^k - (1 - \sigma^k) \sum_{i \neq j} \lambda_{ij}^k \frac{d \ln P_{ij}^k}{d \ln {p_j}^M}$$

where $\lambda_{ij}^k \equiv \frac{P_{ij}^k C_{ij}^k}{\sum_i P_{ij}^k C_{ij}^k}$ is the share of country j's consumption of good k that is produced in i. With a CES demand system, the response of prices P_{ij}^k is

[18] In the terminology of Arkolakis, Costinot and Rodriguez-Clare (2012), this holds true in a CES demand system, and provided sector expenditure shares are constant. Both conditions hold in Sections 2.2 and 2.3.

homogeneous across producing countries i, so that $\frac{d \ln P_{ij}^k}{d \ln p_j{}^M}$ takes identical values for all i.

Substituting back into equation (15), the *macroeconomic* trade elasticity in country j becomes

$$\eta_j = \sum_k \sum_{i \neq j} w_{ij}^k m_j^k \left[\varepsilon^k - (1 - \sigma^k) \frac{d \ln P_{ij}^k}{d \ln p_j{}^M} \lambda_j^k \right] = \sum_k m_j^k (1 - \lambda_j^k) \varepsilon^k \tag{17}$$

Equation (17) illustrates the consequence of a macroeconomic shock: Since relative import prices are all affected, the price elasticity of aggregate imports depends on the share of imports in each sector, denoted with λ_j^k. Thus, the macroeconomic trade elasticity implied by the unconstrained, sector-specific estimates of ε^k is given by η_j in equation (17). The macroeconomic trade elasticity implied by estimates of ε^k that are constrained to homogeneity is given by $\bar{\varepsilon} \sum_k m_j^k (1 - \lambda_j^k)$, and the macroeconomic elasticity implied by an estimate of ε on aggregate data is given by $\hat{\varepsilon} \sum_k m_j^k (1 - \lambda_j^k)$.[19]

II. Results

This section presents the results, starting with a description of the data. Sector-level estimates of ε^k are then discussed and compared with the literature. They are used to compute the macroeconomic trade elasticity η_j, which is compared to what is implied by pooled sectoral data, and by aggregate data.

A. Data

The gravity estimation requires information on bilateral market shares s_{ij}^k and the corresponding bilateral tariffs τ_{ij}^k. Data on the value of bilateral trade flows are obtained from ComTrade, and serve to compute bilateral market shares. Bilateral trade is reported by the importing country and converted into US dollars at current nominal exchange rate. Trade flows are directly aggregated to compute aggregate market shares s_{ij}. Data on bilateral tariffs come from UNCTAD-TRAINS. Observed tariffs are defined as $\tau_{ij}^k - 1$, and aggregate tariffs are computed as a simple average across sectors.[20] Both tariffs and trade flows are collected at the six-digit level of the Harmonized System for each importing country, and aggregated up to the 21 sectors used in the OECD STAN dataset, close to ISIC (rev. 3). Data are collected in 1993 for 16 countries.[21] The gravity regression is identi-

[19] Since by definition the o^k average to zero.

[20] Using the geometric weighted average implied by theory makes little difference to the results. But it involves the import weights m_j^k, which may be systematically large when tariffs are low, and thus over-represent sectors with low tariffs.

[21] The 16 countries are Argentina, Australia, Canada, Chili, China, Colombia, the European Union, India, Indonesia, Japan, Korea, New Zealand, Norway, Switzerland, Thailand and the United States.

fied in the bilateral dimension of trade and tariffs: In order not to over-represent countries with many trade partners, the sample is computed on country-pairs that involve one specific partner, the European Union.[22]

Feenstra's (1994) estimation requires information on US expenditure shares by sector and country of origin, s_{it}^k, the corresponding prices P_{it}^k, and the weights used in aggregation, m_{it}^k. The expenditure shares s_{it}^k and the weights m_{it}^k are computed directly from disaggregated, multilateral trade data released by the Centre d'Etudes Prospectives et d'Informations Internationales (CEPII). Prices are approximated with unit values from the same source, with values in USD at current exchange rates, and quantities in tons.[23] Since outliers are notoriously frequent in unit values, the data are subjected to sampling: In each sector, annual variations in prices and market shares that exceed five times the median are eliminated. Since the cross-section of exporters is what ultimately achieves identification (with instruments), a minimum of 20 exporters is imposed for each HS6 good over the whole observed time period. The end sample represents 77 percent of the total value of US imports. The data are available at the 6-digit level of the harmonized system (HS6), and cover around 5,000 products over the 1996-2004 period for a large cross-section of countries. The estimation is performed in deviations from a reference exporter. In sectoral estimates, this is the first exporting country (in alphabetical order) that is present in the US over the period. In aggregate data, it is Canada.

To correct for the response of price indices to a macroeconomic shock, a measure λ_j^k of the share of imports in country j's total expenditures on good k is required. The import shares are obtained from the US input-output tables, available at the ISIC (Rev. 3) level. They are computed as the 1997 ratio of imports over domestic gross output.[24]

B. Sector Estimates

This section presents the elasticity estimates obtained at sector-level with both approaches. Figure 1 reports the values of ε^k implied by the gravity estimation for 21 sectors. They range from zero to -41.8, with average -11.4 and median -9.6. The distribution of estimates is similar to Table 1 in Caliendo and Parro (2012), with similar moments and extrema.

Figure 2 reports trade elasticity estimates for 56 ISIC sectors using Feenstra (1994). They range between -2.2 and -29, with mean -5.4 and median -3.9.

Following Caliendo and Parro (2012), the countries are chosen on grounds of the reliability of their tariff data, and the variety of trade partners available in each sector. When bilateral tariffs are not available in 1993, they are replaced by the value closest in time over the four previous or three subsequent years.

[22]Head and Mayer (2014) recommend using a reference country in gravity regressions. For completeness, results are also discussed when the sample is computed on all pairs of countries (i, j) in the sample, as in Caliendo and Parro (2012).

[23]See Gaulier and Zignago (2010) for a description of the database.

[24]Results are virtually identical if the import values at the numerator of λ_j^k are directly taken from ComTrade data, and the denominator is a measure of total domestic output taken from OECD STAN.

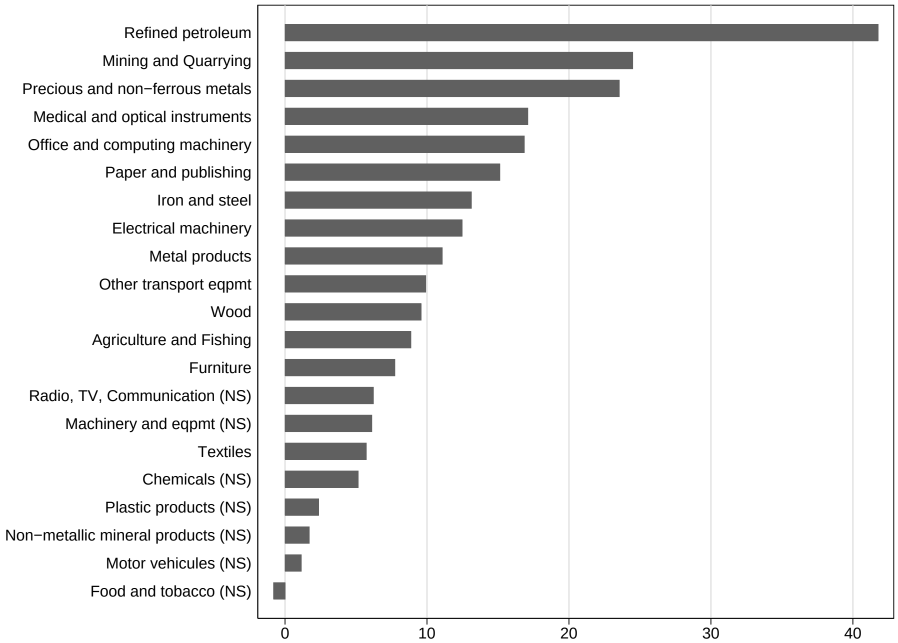

Figure 1. Estimates of the tariff elasticity of imports based on Caliendo and Parro (2012).

Note: The figure plots minus the gravity estimates, by sector. (NS) indicates non-significance at the 10% level.

Broda and Weinstein (2006) implement a similar estimation on 243 SITC-3 sectors between 1972 and 1988, with mean −4.9, and median −2.0. Their mean estimate is close to the one presented here. Their median estimate is closer to zero than ours, reflecting the differences in classifications in both exercises.[25]

Are the sector-level estimates in this paper comparable with those obtained in the literature more generally? Houthakker and Magee (1969) report in Table 6 a median import price elasticity in manufactures estimated at −4.05. This is virtually identical to the median value obtained here across 56 manufacturing sectors. Similarly, Kreinin (1967) documents an elasticity for manufactures equal to −4.71. Interestingly, both papers make use of information on the prices of domestic goods to estimate the Armington elasticity. This paper does not, and it is reassuring that the estimates obtained with either data should be close.

Both sets of estimates confirm that sector-level elasticities are heterogeneous and relatively high on average. There is little doubt that such is the case in the literature. Romalis (2007) estimates elasticities between −3 and −12 at the HS6

[25]The gravity estimation yields large elasticity values, just like the meta-analysis in Head and Mayer (2014): Their Table 5 indicates that gravity estimates are systematically larger in absolute value than those obtained if changes in import prices are instrumented with changes in wages, in observed prices, or in exchange rates.

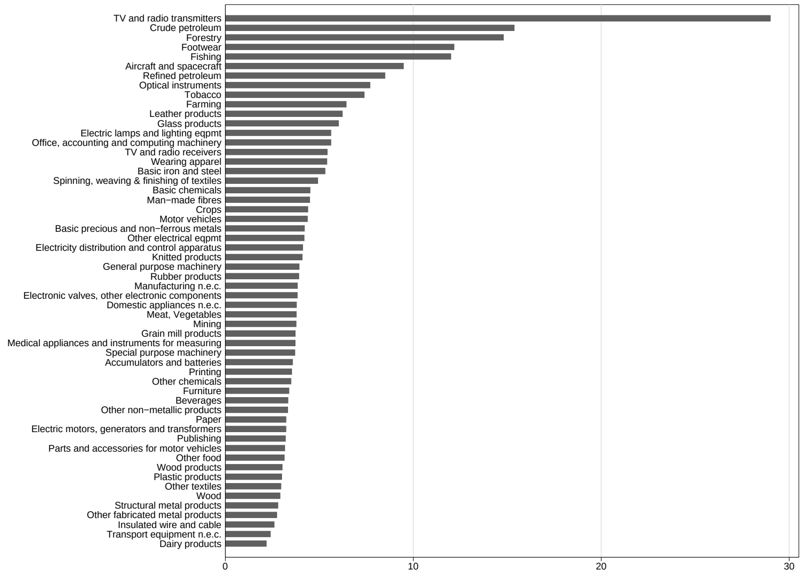

FIGURE 2. ESTIMATES OF THE ARMINGTON ELASTICITY BASED ON FEENSTRA (1994)

Note: The figure plots the value of substitution elasticities $(1 - \varepsilon^k)$ obtained with Feenstra's (1994) methodology. The elasticity is obtained using a grid search procedure when the IV strategy implies parameters that are not consistent with the model.

level. Head and Ries (2001) report values between −6.9 and −10.4 at the 3-digit SIC level. Hummels (2001) obtains values between −2 and −7 in two-digit data. A key contention in this paper is that sectoral estimates are high on average, and that they decrease with the level of aggregation. This has to be true if a heterogeneity bias is to explain the elasticity puzzle. The claim is supported by the meta-analysis in Disdier and Head (2008), who consider estimates of the effect of distance on trade flows. They find an index capturing the level of disaggregation is systematically positive, so that average sectoral estimates fall with the level of aggregation.[26] Hummels (2001) finds estimates of the elasticity fall from −3.8 to −7.3 as aggregation moves from one- to four-digit. Between 1972 and 1988, Broda and Weinstein (2006) report an average elasticity of −4.9 in SITC3 data with 243 categories, falling to −10.7 in TSUSA/HTS data with 12,219 categories.

Are there elasticity estimates that are low in sectoral data? Reinert and Roland-Holst (1992) or Blonigen and Wilson (1999) report elasticities for more than 150 sectors, with only few values below −2. Gallaway, McDaniel and Rivera (2003) consider 309 US sectors, and find elasticities between −0.5 and −4, averaging −0.55. These are low averages. However, in all three papers the price of imported

[26] The dispersion in distance coefficients can stem either from heterogeneous import elasticities, or from the heterogeneous effects of distance on trade costs.

Table 1—Aggregate, constrained and unconstrained elasticities

	Caliendo-Parro	Feenstra
Aggregate elasticity	-1.790***	-2.001***
	(0.426)	(0.116)
Constrained elasticity	-2.375***	-2.005***
	(0.506)	(0.150)
Unconstrained elasticity	-5.639***	-4.174***
	(1.171)	(0.106)

Note: Standard errors in parentheses, *** denotes significance at the 1 percent level. Import elasticity computed as $\eta_j = \sum_k m_j^k(1-\lambda_j^k)\varepsilon_j^k$, $\eta_j = \bar{\varepsilon}\sum_k m_j^k(1-\lambda_j^k)$ and $\eta_j = \hat{\varepsilon}\sum_k m_j^k(1-\lambda_j^k)$, in the unconstrained, constrained and aggregate cases, respectively.

goods relative to domestic substitutes is not instrumented. Inasmuch as the price of domestic goods falls in response to the threat of foreign competition, this creates an attenuating endogeneity bias, which can explain such low values. The two estimators used in this paper were designed to alleviate such endogeneity.[27]

C. Aggregate and Constrained Estimates

Table 1 reports the values of the trade elasticity in response to a macroeconomic shock, as implied by either the gravity approach, or by Feenstra's estimation. For either approach, the table first reports the macroeconomic elasticity implied by aggregate data, $\hat{\varepsilon}\sum_k m_j^k(1-\lambda_j^k)$, then its value implied by pooled sectoral data, $\bar{\varepsilon}\sum_k m_j^k(1-\lambda_j^k)$, and finally its value implied by heterogeneous sectoral data, $\sum_k m_j^k(1-\lambda_j^k)\varepsilon^k$.

In aggregate data, the macroeconomic trade elasticity is -1.79 using the gravity estimation, and -2.00 using Feenstra's estimation. The point estimates are not significantly different from each other. They are not significantly different either from conventional estimates obtained on aggregate data. For instance, they are within the range reported in Francis, Schumacher and Stern (1976), or in Goldstein and Kahn (1985). For US aggregate trade, the latter report a range of -1.03 to -1.76. In the words of Backus, Kehoe and Kydland (1994): "The most reliable studies seem to indicate that for the United States the elasticity [of substitution] is between 1 and 2" (p.91). This corresponds to an aggregate import elasticity around -1.

Are there elasticity estimates that are high in aggregate data? Eaton and Kortum (2002) identify the aggregate trade elasticity that, in a Ricardian trade model, maps into the international distribution of productivity. They find $\varepsilon =$

[27]Table 5 in Head and Mayer (2014) reports similar results: In a meta-analysis of gravity regressions where changes in the relative price of imports are captured with tariffs, the median trade elasticity is -5.09. But it is -1.12 if import prices are measured with exchange rate movements, wage differences, or with directly observed (and non-instrumented) changes in import prices.

-8.28, a value comparable to conventional microeconomic estimates. Simonovska and Waugh (2014) argue this estimate is large because of an approximation in the measure of trade costs. They find an aggregate trade elasticity closer to -4. Bernard et al. (2003) use a simulated method of moments in an analogous Ricardian model of trade, and find an elasticity of -3.6. Both approaches estimate ε, rather than the macroeconomic elasticity η, because they are not concerned with the consequences of uniform, aggregate shocks. In the US, the correcting factor $\sum_k m_j^k(1-\lambda_j^k)$ brings these aggregate elasticity estimates in the same range as the ones presented in Table 1.[28]

The second row of Table 1 reports the macroeconomic trade elasticities implied by sector panel estimates constrained to homogeneity. The gravity approach implies a value of -2.38, larger in absolute value than the aggregate elasticity, but not significantly so. The discrepancy is small and insignificant, which suggests the differences between the hypotheses that underpin Theorems 1 and 2 are negligible in the data.[29] Feenstra's approach implies a constrained elasticity equal to -2.00, virtually identical to the aggregate estimate. This suggests the different hypotheses that underpin Theorems 3 and 4 are fulfilled in the data.[30]

The last row in Table 1 reports the trade elasticity implied by heterogeneous sectoral data, $\sum_k m_j^k(1-\lambda_j^k)\varepsilon^k$. In both cases it is significantly larger in absolute value than either the constrained or the aggregate elasticity: -5.64 in the gravity setup, and -4.17 using Feenstra. These significant discrepancies illustrate how a heterogeneity bias can explain the elasticity puzzle: In the same estimation, exploiting the same dimension of the same data, constraining trade elasticities to homogeneity across sectors results in an estimate that is close to zero, much as estimates from aggregate data are.[31]

Where do these biases come from? The heterogeneity bias in the gravity regression comes from a negative relation between sector elasticities and the cross-country dispersion in tariffs $var_\ell(\ln \tilde{\tau}_\ell^k)$: In the data, the correlation is equal to -0.305. This tends to push the estimated values of both $\bar{\varepsilon}$ and $\hat{\varepsilon}$ towards zero, the heterogeneity bias. The correlation between ε^k and m^k is also negative, but low, at -0.162. This has minimal effects on end estimates, since $\hat{\varepsilon}$ is not significantly different from $\bar{\varepsilon}$.[32]

[28] Ruhl (2008) also refers to Baier and Bergstand (2001), who estimate an elasticity of substitution equal to 6.43 on aggregate data. But their confidence interval ranges from 2.44 to 10.42.

[29] The aggregate market share is obtained by directly summing sector-level trade flows, so that import weights are effectively allowed to be different across countries.

[30] The instruments are computed at country-level for both the aggregate and the sector-level panel regressions. If instead sector-specific instruments are used in the panel regression, the estimate of the constrained elasticity falls to -2.23. Aggregate estimates are computed holding import weights constant over time, at their initial value, but letting them vary across countries. If observed import weights are used instead, the aggregate import elasticity falls to -2.29.

[31] The gravity approach implies seven elasticities insignificantly different from zero at 10 percent confidence level, that are reported in Figure 1. They are set to zero when computing $\sum_k m_j^k(1-\lambda_j^k)\varepsilon^k$. If the point estimates are kept instead, the elasticity falls to -6.57.

[32] When the gravity regression is performed instead on a sample of *all* country pairs, the aggregate elasticity is -1.250 (0.233), the constrained elasticity is -1.980 (0.299), and the unconstrained elasticity is -4.526 (0.457). The correlation between ε^k and $var_\ell(\ln \tilde{\tau}_\ell^k)$ is -0.25. Then, there are five estimates

In Feenstra's approach, the heterogeneity bias comes from a negative relation between sector elasticities and the dispersion across countries in the variance of prices conditional on supply shocks, $var_i(\sigma^2_{\delta_i^k})$. The conditional variance can be inferred from the estimates of the fundamental parameters of the model, along with the system formed by equations (8) and (9). In the data, the correlation is equal to -0.134, which tends to push the values of both $\bar{\varepsilon}$ and $\hat{\varepsilon}$ towards zero, the heterogeneity bias. In addition, the correlation between ε^k and m^k is slightly positive, at 0.0525. This affects the value of $\hat{\varepsilon}$ minimally, and not in a way that is observable in end estimates.[33] The irrelevance of the covariance between sector trade elasticities and sector openness in both estimators emphasizes that the paper's results do pertain to a heterogeneity bias, rather than to a composition effect.

As discussed in Appendix C, heterogeneity in the elasticity of supply ω^k has ambiguous consequences on the magnitude of the heterogeneity bias as implied by Feenstra (1994): Estimates of $\bar{\varepsilon}$ (and $\hat{\varepsilon}$) are also biased if there exists a systematic relation between ω^k and $var_i(\sigma^2_{\xi_i^k})$, the dispersion across countries in the variance of prices conditional on demand shocks. In the data, the correlation is equal to -0.112, which means heterogeneity in ω^k may affect the estimates of $\bar{\varepsilon}$ (and $\hat{\varepsilon}$). To gauge how much it does, the expressions for $cov_{ikt}(\bar{X}^k_{1i}, u^k_{it})$ and $cov_{ikt}(\bar{X}^k_{2i}, u^k_{it})$ in Appendix C.1.2 are evaluated using the estimates of the model's parameters and structural shocks. They reflect the magnitude of the bias when both sources of heterogeneity, ε^k and ω^k, are active. Their values are computed both in general, and under the constraint that $\omega^k = \omega$. The corresponding trade elasticities are inferred accordingly: with both sources of heterogeneity, the constrained elasticity is equal to -2.38; with heterogeneity in ε^k only, it is equal to -2.02. The difference reflects the importance of heterogeneous ω^k.

III. Calibrating the Elasticity

What economic difference does it make to have a macroeconomic trade elasticity equal to -1.75 or to -5? In conventional macroeconomic models where the trade elasticity maps into the Armington elasticity, the consequences can be important. For instance, with a trade elasticity of -5 (i.e. an elasticity of substitution of 6), the exchange rate depreciation necessary for a 6 percent US current account deficit to disappear is shaved by one third relative to the headline result in Obstfeld and Rogoff (2005). In Cole and Obstfeld (1991), the Armington elasticity is unitary, which implies terms of trade movements deliver perfect insurance

of ε^k that are not significantly different from zero at 10 percent confidence level, which are set to zero to obtain the unconstrained elasticity of -4.526. When they are not, the estimate drops to -4.664.

[33] The Feenstra estimation is implemented on 56 ISIC (Revision 3) industries, because the data needed to aggregate sector elasticities are not available at the HS6 level. Such coarse level of estimation can effectively impose a homogeneity constraint between all HS6 products within an ISIC category, which can create a heterogeneity bias. As a robustness check, elasticities were estimated for all HS6 goods, and still aggregated using weights at ISIC level. The main results were unchanged. This also takes care of concerns about outliers in the estimates of ε^k in Figure 2.

against country-specific shocks. When it is not unitary, financial diversification becomes of the essence: for values of the Armington elasticity below one, there is a home bias in portfolio holdings (see Heathcote and Perri, 2013); for values of the Armington elasticity above one, there is a preference for foreign assets (see Coeurdacier, 2009). By analogy, the role of monetary policy also depends on the substitutability between domestic and foreign goods: with non-unitary elasticity, terms of trade shocks are not automatically insured, the open economy matters for welfare, and thus for monetary policy (see De Paoli, 2009).

The findings of this paper are important as well from the standpoint of the modeling of heterogeneity. In a world of sectoral heterogeneity, a multi-sector model is desirable but not always tractable: Most models in macroeconomics have one sector only. A one-sector version is a convenient shortcut, provided its predictions are equivalent to what the calibration of a hypothetical multi-sector model would imply. The shortcut is often used, but the equivalence does not always hold. The rest of this section illustrates this possibility in two classic models, in macroeconomics and in trade. The first one is the workhorse model of international business cycles, due to Backus, Kehoe and Kydland (1994) [BKK henceforth]. The trade elasticity defines the Armington elasticity between foreign and domestic goods. The second one is the generalized model of international trade in Arkolakis, Costinot and Rodriguez-Clare (2012) [ACR henceforth], where the trade elasticity can be interpreted either as a supply or as a preference parameter.

A. The J-Curve in BKK

BKK introduce a dynamic model of the J-curve, aimed at reproducing the cross-correlation between the terms of trade and net exports. Appendix E reviews the details and calibration of a simple four-sector extension of BKK. The sectors differ in the value of the Armington elasticity, $1-\varepsilon^k$. Technology shocks are aggregate. The model is used to generate predictions on the dynamic correlation of the terms of trade with net exports when sector-level elasticities of substitution are calibrated to heterogeneous values.

Figure 3 plots the cross-correlograms between net exports and the terms of trade for various setups. Both variables are simulated in response to a domestic technology shock that occurs at time t. The plain thick line corresponds to the four-sector model, where sectoral Armington elasticities are calibrated to the vector (2.9, 3.6, 4.5, 10.5). The four values are chosen to replicate the quartiles of the distribution of estimates of ε^k, obtained with Feenstra's approach. The cross-correlogram displays the well known J shape in the response of net exports.

Figure 3 also reports J-curve estimates implied by the one-sector version of BKK. The dotted thin line corresponds to the cross-correlogram implied by an Armington elasticity calibrated to $1-\hat{\varepsilon}\sum_k m_j^k(1-\lambda_j^k)=3.00$. The dotted thick line traces the cross-correlogram implied by $1-\sum_k m_j^k(1-\lambda_j^k)\varepsilon^k=5.17$. It is clear from the figure that only the unconstrained estimate replicates the J-curve implied by the multi-sector version of BKK. Calibrating instead the Armington

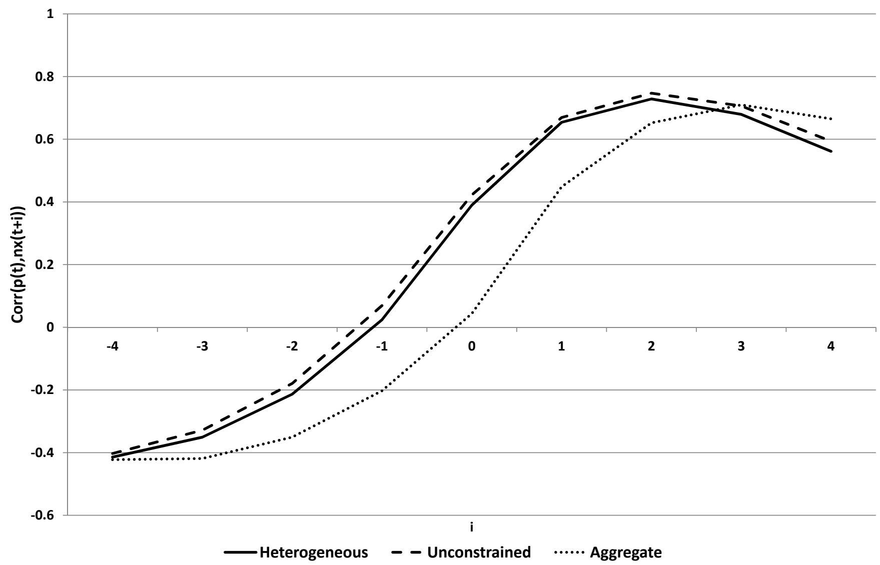

Figure 3. The J-curve in a four-sector version of BKK.

Note: Cross-correlograms between net exports and the terms of trade in the 4-sector BKK model ("Heterogeneous" curve) and the one-sector model ("Unconstrained" and "Aggregate" lines) in response to an aggregate productivity shock at time t. The "Heterogeneous" version is calibrated with a vector of elasticities equal to $(2.9; 3.6; 4.5; 10.5)$. The "Unconstrained" case is calibrated with an elasticity of 5.17. The "Aggregate" case is calibrated with an elasticity of 3.0.

elasticity to aggregate data generates a J-curve that is at odds with the predictions of a heterogeneous multi-sector model. The J-curve implied by the constrained elasticity is virtually identical.

It is well known that BKK has counterfactual implications with high Armington elasticity. A low value is necessary to reproduce the negative contemporaneous correlation between the terms of trade and net exports that is observed in the data. Figure 3 confirms this fact: a J-curve implied by an Armington elasticity of 3 displays correlation closer to zero at time t than if it is calibrated at 5.17. The paper does not propose to reconcile with aggregate data the J-curve implied by a multi-sector version of BKK. Rather, the paper emphasizes that estimates of the Armington elasticity from aggregate data actually ignore sectoral heterogeneity. Figure 3 suggests this is the reason why a calibration of BKK on aggregate data is unable to reproduce the J-curve implied by a multi-sector version of BKK.

B. The Welfare Gains from Trade in ACR

ACR show that the welfare gains from trade can be summarized identically across a wide class of models. All that is needed is an estimate of the trade elasticity ε, and the share of imports in consumption λ_j. The formula is valid in a broad range of models, including those that underpin the two empirical approaches in this paper. It holds in one- or multi-sector versions of each model. In a multi-sector model, ACR show that

$$d\ln W_{MS} = \sum_k \frac{\alpha_j^k}{\varepsilon^k} \left[d\ln(1-\lambda_j^k) + I^k \; d\ln\left(\frac{E_j^k}{R_j^k}\right)\right] \tag{18}$$

where $d\ln W_{MS}$ denotes the welfare gains from trade in a multi-sector model, α_j^k are expenditure shares, and I^k is an indicator variable taking value zero under perfect competition and one otherwise. The difference $d\ln(1-\lambda_j^k) = \ln(1-\lambda_j^k) - 0$ is measured between observed openness and autarky. As discussed in ACR the gains from trade predicted by multi-sector gravity models under monopolistic competition differ from those predicted under perfect competition because of scale effects. Those effects are captured by the second term in equation (18). E_j^k is the share of sector k in expenditures (which can be different from α_j^k because the model allows for intermediate goods). R_j^k is the share of sector k in revenue, which is equal to one under autarky.

If all sectors have identical trade elasticities, equation (18) simplifies into

$$\begin{aligned} d\ln W_{MS} &= \frac{1}{\varepsilon}\sum_k \alpha_j^k \left[d\ln(1-\lambda_j^k) + I^k d\ln\left(\frac{E_j^k}{R_j^k}\right)\right] \\ &= \frac{d\ln(1-\lambda_j)}{\varepsilon} = d\ln W_{OS} \end{aligned} \tag{19}$$

where $d\ln W_{OS}$ denotes the welfare gains from trade in country j, as implied by a one-sector model. Since they effectively stem from the same theory, equations (18) and (19) must imply identical welfare gains from trade.[34] This paper conjectures that the equality in equation holds only if the one-sector trade elasticity ε is calibrated using an estimate that reflects sector-level heterogeneity. Table 2 reports the values of $d\ln W_{MS}$ corresponding to the estimates of ε^k obtained in section 3.2, and the values of $d\ln W_{OS}$ corresponding to the estimates in section 3.3.[35] All other calibrated values are taken from Costinot and Rodriguez-Clare

[34]The simplification builds from the definition of $1-\lambda_j = \frac{\sum_k P_{jj}^k \; C_{jj}^k}{\sum_k P_j^k \; C_j^k} = \sum_k \frac{P_{jj}^k \; C_{jj}^k}{P_j^k \; C_j^k} \frac{P_j^k \; C_j^k}{\sum_k P_j^k \; C_j^k}$ $= \sum_k (1-\lambda_j^k)\alpha_j^k$. Simple differentiation implies $d\ln(1-\lambda_j) = \frac{\sum_k \alpha_j^k \; d(1-\lambda_j^k)}{1-\lambda_j} = \sum_k \frac{1-\lambda_j^k}{1-\lambda_j} \; \alpha_j^k \; d\ln(1-\lambda_j^k) = \sum_k \alpha_j^k \; d\ln(1-\lambda_j^k)$ since in a one-sector model $\lambda_j^k = \lambda_j$ for all k. The second term of equation (18) disappears, whatever the market structure, under the assumption that goods trade is balanced. This is different from Ossa (2012), who accommodates heterogeneity in both ε^k and λ_j^k. This paper focuses on the heterogeneity in ε^k.

[35]The results in ACR hold in general equilibrium, where price effects are accounted for, as testified by the presence of λ in equations (18) and (19). The calibration is therefore performed using estimates of ε^k and ε.

Table 2—Percentage welfare gains from trade in the one- and multiple-sector versions of ACRC

	Caliendo-Parro	Feenstra
Multiple-sector, Heterogeneous elasticities		
Perfect competition	1.27	
	(0.043)	
Monopolistic competition	0.55	0.79
	(0.022)	(0.001)
One-sector, Unconstrained elasticity	0.71	0.97
	(0.012)	(0.002)
One-sector, Aggregate elasticity	2.30	2.05
	(0.016)	(0.004)

Note: Standard errors in parentheses. One-sector welfare gains computed from: $d \ln W_{OS} = \frac{\ln(1-\lambda_j)}{\varepsilon}$ where ε is equal to $\hat{\varepsilon}$ in the "Aggregate" case and $\sum m^k \varepsilon^k$ in the "Unconstrained" case. Multiple-sector welfare gains computed from $d \ln W_{MS} = \sum_k \frac{\alpha_j^k}{\varepsilon^k} \ln(1 - \lambda_j^k)$. The calibration of α_j^k, λ_j, λ_j^k, E_j^k and R_j^k uses US IO data following Costinot and Rodriguez-Clare (2014).

(2014). Since Feenstra's estimation assumes monopolistic competition, only those results are presented (see Simonovska and Waugh, 2014).

The first column of Table 2 reports the welfare gains implied by the gravity approach. These elasticity estimates are valid in all the models considered in ACR. With heterogeneous sectors, $d \ln W_{MS} = 0.55$ percent under monopolistic competition, and 1.27 percent, in perfect competition. Welfare gains $d \ln W_{OS}$ are of the same order, equal to 0.71 percent, when the calibration is done using heterogeneous elasticities. In contrast, a one-sector model calibrated with a trade elasticity estimated from aggregate data implies $d \ln W_{OS} = 2.30$ percent, three times larger than $d \ln W_{OS}$ calibrated with ε, and between two and five times larger than $d \ln W_{MS}$.[36]

The second column of the Table reports the welfare gains implied by estimates from Feenstra (1994). With heterogeneous sectors and monopolistic competition, $d \ln W_{MS} = 0.79$ percent. The one-sector model calibrated with an unconstrained elasticity gives $d \ln W_{OS} = 0.97$ percent, but if it is calibrated with an elasticity estimated from aggregate data, the welfare gains jump to $d \ln W_{OS} = 2.05$ percent, more than twice the multi-sector value.

Both sets of results confirm the implications of an estimate from aggregate data that ignores sectoral heterogeneity. Here it has implications on the welfare gains from trade. It should be clear that the point of this exercise is not to settle the question of the welfare gains from trade: For instance, it is not surprising that the gains should be small in the US, a relatively closed economy. Instead, the exercise

[36]The reported values of $d \ln W_{MS}$ include sectors with an insignificant elasticity estimate. If these are dropped instead, the four estimates in Table 2 become: 2.30, 0.71, 0.32 and 0.16, an even larger discrepancy.

is meant to emphasize the importance of a calibrated parameter that accounts for sector-level heterogeneity, or not.

IV. Conclusion

In absolute value, trade elasticity estimates have been found to decrease with the level of aggregation. This paper argues the finding is a manifestation of a heterogeneity bias: Aggregate data constrain away microeconomic heterogeneity in trade elasticities. The constraint pushes estimates towards zero because prices typically change in sectors with inelastic trade. Thus, aggregate price changes are associated with little response in quantities, even though some -potentially large- sectors are actually elastic. The argument is established theoretically for two prominent estimators of trade elasticities: one based on the ubiquitous gravity model, the other based on a CES demand system. It is verified in US data, where the same data yield a macroeconomic trade elasticity around -1.75 in the aggregate, but around -5 on average across sectors. The discrepancy is commensurate with the elasticity puzzle.

The fact the elasticity puzzle can be explained by a homogeneity constraint has far-ranging implications. Calibrating a one-sector model using a trade elasticity estimate obtained from aggregate data is likely to result in predictions that fail to reflect sector-level heterogeneity, just as the estimate itself does. Instead, a one-sector model ought to be calibrated using a weighted average of sector-level estimates, the unconstrained elasticity introduced in this paper, if it is to replicate the predictions of its multi-sector counterpart. The paper shows this to be the case in two important models in international economics.

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Details on the heterogeneity bias in Caliendo and Parro (2012)

A1. Pooled microeconomic estimate

The heterogeneity bias depends on the covariance between the explanatory variable and the residual, computed over country triplets *and* sectors:

$$\begin{aligned} cov_{\ell k}(\ln\tilde{\tau}_\ell^k, u_\ell^k) &= \frac{1}{KL}\sum_\ell\sum_k\left[\ln\tilde{\tau}_\ell^k - E_{\ell k}(\ln\tilde{\tau}_\ell^k)\right]\left[u_\ell^k - E_{\ell k}(u_\ell^k)\right] \\ &= -\frac{1}{KL}\sum_k\sum_\ell o^k\left(\ln\tilde{\tau}_\ell^k - E_\ell(\ln\tilde{\tau}_\ell^k)\right)\left(\ln\tilde{\tau}_\ell^k - E_\ell(\ln\tilde{\tau}_\ell^k)\right) \\ &- \frac{1}{K}\sum_k\left(E_\ell(\ln\tilde{\tau}_\ell^k) - E_{\ell k}(\ln\tilde{\tau}_\ell^k)\right)\left(E_\ell(o^k\ln\tilde{\tau}_\ell^k) - E_{\ell k}(o^k\ln\tilde{\tau}_\ell^k)\right) \\ &= -\frac{1}{K}\sum_k o^k var_\ell(\ln\tilde{\tau}_\ell^k) - cov_k(E_\ell(o^k\ln\tilde{\tau}_\ell^k), E_\ell(\ln\tilde{\tau}_\ell^k)) \end{aligned}$$

where L is the number of country triplets in the sample, and K is the number of sectors. The second equality uses the definition of $u_\ell^k = e_\ell^k - o^k\ln\tilde{\tau}_\ell^k$, along with Assumption 1.

A2. Aggregate estimate

The covariance term $cov_\ell(\ln\tilde{\tau}_\ell, u_\ell)$ can be rewritten:

$$\begin{aligned} cov_\ell(\ln\tilde{\tau}_\ell, u_\ell) &= \frac{1}{L}\sum_\ell\left[\left(\sum_k m^k[\ln\tilde{\tau}_\ell^k - E_\ell(\ln\tilde{\tau}_\ell^k)]\right)\left(\sum_k m^k[u_\ell^k - E_\ell(u_\ell^k)]\right)\right] \\ &= -\frac{1}{L}\sum_\ell\left[\left(\sum_k m^k[\ln\tilde{\tau}_\ell^k - E_\ell(\ln\tilde{\tau}_\ell^k)]\right)\left(\sum_k m^k o^k[\ln\tilde{\tau}_\ell^k - E_\ell(\ln\tilde{\tau}_\ell^k)]\right)\right] \\ &= -\sum_k {m^k}^2 o^k var_\ell(\ln\tilde{\tau}_\ell^k) - \sum_k\sum_{k'\neq k} m^k m^{k'} o^k cov_\ell(\ln\tilde{\tau}_\ell^k, \ln\tilde{\tau}_\ell^{k'}) \end{aligned}$$

where L is the number of country triplets in the sample, and $u_\ell^k \equiv e_\ell^k - o^k\ln\tilde{\tau}_\ell^k$. The first line uses assumption 4 that weights are homogenous across countries. The second line uses assumption 3 that the sector-level residual is orthogonal to tariffs within and between sectors.

Details on the estimation of trade elasticities in Feenstra (1994)

Following Feenstra (1994), the structural elasticities derived from the ψ_1^k and ψ_2^k coefficients estimated in equation (10) rewrite:

$$\hat{\varepsilon}^k = \frac{\hat{\psi}_2^k + \sqrt{{\hat{\psi}_2^k}^2 + 4\hat{\psi}_1^k}}{-2\hat{\psi}_1^k}, \qquad \hat{\omega}^k = -\hat{\varepsilon}^k \hat{\psi}_1^k$$

They are theoretically consistent if and only if $\hat{\psi}_1^k > 0$. When $\hat{\psi}_1^k < 0$, we follow Broda and Weinstein (2006) and implement a search algorithm that minimizes the sum of squared residuals in equation (10) over the intervals of admissible values for the supply and demand elasticities (i.e. for $\varepsilon^k < 0$ and $\omega^k > 0$). The corresponding standard errors are obtained via bootstrapping the procedure in 1,000 repetitions.

When estimates of ψ_1^k and ψ_2^k are in the permissible range, the variances of $\hat{\varepsilon}^k$ and $\hat{\omega}^k$ are computed using the second-order moments of $\hat{\psi}_1^k$ and $\hat{\psi}_2^k$ and a first-order approximation of the parameters around their true value:

$$\begin{aligned} \omega^k &= \hat{\omega}^k + \left.\frac{\partial \omega^k}{\partial \varepsilon^k}\right|_{\varepsilon^k=\hat{\varepsilon}^k} (\varepsilon^k - \hat{\varepsilon}^k) + \left.\frac{\partial \omega^k}{\partial \psi_1^k}\right|_{\psi_1^k=\hat{\psi}_1^k} (\psi_1^k - \hat{\psi}_1^k) \\ \varepsilon^k &= \hat{\varepsilon}^k + \left.\frac{\partial \varepsilon^k}{\partial \psi_1^k}\right|_{\psi_1^k=\hat{\psi}_1^k} (\psi_1^k - \hat{\psi}_1^k) + \left.\frac{\partial \varepsilon^k}{\partial \psi_2^k}\right|_{\psi_2^k=\hat{\psi}_2^k} (\psi_2^k - \hat{\psi}_2^k) \end{aligned}$$

which implies:

$$\begin{aligned} var(\hat{\omega}^k) &= \left(\left.\frac{\partial \omega^k}{\partial \varepsilon^k}\right|_{\varepsilon^k=\hat{\varepsilon}^k}\right)^2 var(\hat{\varepsilon}^k) + 2\left.\frac{\partial \omega^k}{\partial \varepsilon^k}\right|_{\varepsilon^k=\hat{\varepsilon}^k} \left.\frac{\partial \omega^k}{\partial \psi_1^k}\right|_{\psi_1^k=\hat{\psi}_1^k} cov(\hat{\varepsilon}^k, \hat{\psi}_1^k) \\ &\quad + \left(\left.\frac{\partial \omega^k}{\partial \psi_1^k}\right|_{\psi_1^k=\hat{\psi}_1^k}\right)^2 var(\hat{\psi}_1^k) \\ var(\hat{\varepsilon}^k) &= \left(\left.\frac{\partial \varepsilon^k}{\partial \psi_1^k}\right|_{\psi_1^k=\hat{\psi}_1^k}\right)^2 var(\hat{\psi}_1^k) + 2\left.\frac{\partial \varepsilon^k}{\partial \psi_1^k}\right|_{\psi_1^k=\hat{\psi}_1^k} \left.\frac{\partial \varepsilon^k}{\partial \psi_2^k}\right|_{\psi_2^k=\hat{\psi}_2^k} cov(\hat{\psi}_1^k, \hat{\psi}_2^k) \\ &\quad + \left(\left.\frac{\partial \varepsilon^k}{\partial \psi_2^k}\right|_{\psi_2^k=\hat{\psi}_2^k}\right)^2 var(\hat{\psi}_2^k) \\ cov(\hat{\varepsilon}^k, \hat{\psi}_1^k) &= \left(\left.\frac{\partial \varepsilon^k}{\partial \psi_1^k}\right|_{\psi_1^k=\hat{\psi}_1^k}\right) var(\hat{\psi}_1^k) + \left(\left.\frac{\partial \varepsilon^k}{\partial \psi_2^k}\right|_{\psi_2^k=\hat{\psi}_2^k}\right) cov(\hat{\psi}_1^k, \hat{\psi}_2^k) \end{aligned}$$

where: $\frac{\partial\omega^k}{\partial\varepsilon^k} = -\psi_1^k$, $\frac{\partial\omega^k}{\partial\psi_1^k} = -\varepsilon k$, $\frac{\partial\varepsilon^k}{\partial\psi_1^k} = \frac{{\psi_2^k}^2+2\psi_1^k+\psi_2^k\sqrt{{\psi_2^k}^2+4\psi_1^k}}{2\,{\psi_1^k}^2\sqrt{{\psi_2^k}^2+4\psi_1^k}}$, $\frac{\partial\varepsilon^k}{\partial\psi_2^k} = \frac{\sqrt{{\psi_2^k}^2+4\psi_1^k}+\psi_2^k}{-2\psi_1^k\sqrt{{\psi_2^k}^2+4\psi_1^k}}$.

$var(\hat{\psi}_1^k)$, $var(\hat{\psi}_2^k)$, and $cov(\hat{\psi}_1^k, \hat{\psi}_2^k)$ are directly obtained from estimates. The standard errors of the aggregate and constrained coefficients $\hat{\varepsilon}$ and $\hat{\omega}$ are estimated following an analogous argument.

Details on the Heterogeneity bias in Feenstra (1994)

C1. *Pooled microeconomic estimate*

The case with homogeneous supply elasticity $\omega^k = \omega$. — Using assumption 5 in equations (11)-(12), it is easy to compute the instruments for equation (13):

$$\begin{aligned}
\bar{X}_{1i}^k &= \frac{1}{T}\sum_t \left[z_s^k(\delta_{it}^k - \delta_{rt}^k) + b_s^k(\xi_{it}^k - \xi_{rt}^k)\right]^2 = {z_s^k}^2\,\sigma^2_{\delta_i^k} + {b_s^k}^2\,\sigma^2_{\xi_i^k} \\
\bar{X}_{2i} &= \frac{1}{T}\sum_t \left[z_P^k(\delta_{it}^k - \delta_{rt}^k) + b_P^k(\xi_{it}^k - \xi_{rt}^k)\right]\left[z_s^k(\delta_{it}^k - \delta_{rt}^k) + b_s^k(\xi_{it}^k - \xi_{rt}^k)\right] \\
&= z_s^k z_P^k \sigma^2_{\delta_i^k} + b_s^k b_P^k \sigma^2_{\xi_i^k}
\end{aligned}$$

where T is the number of years in the sample and $\sigma^2_{\delta_i^k} \equiv var_t(\delta_{it}^k - \delta_{rt}^k)$, $\sigma^2_{\xi_i^k} \equiv var_t(\xi_{it}^k - \xi_{rt}^k)$. This uses the fact that the estimation in pooled microeconomic data includes sector-specific fixed effects, i.e., that $E_{it}(X_{1it}^k) = 0$ and $E_{it}(X_{2it}^k) = 0$.

The residual of the pooled equation writes:

$$\begin{aligned}
u_{it}^k &= -\frac{1}{\varepsilon}(\xi_{it}^k - \xi_{rt}^k)(\delta_{it}^k - \delta_{rt}^k) + \frac{o^k}{\varepsilon}(\delta_{it}^k - \delta_{rt}^k)(d\ln P_{it}^k - d\ln P_{rt}^k) \\
&= -\frac{1}{\varepsilon}(\xi_{it}^k - \xi_{rt}^k)(\delta_{it}^k - \delta_{rt}^k) + \frac{o^k}{\varepsilon}(\delta_{it}^k - \delta_{rt}^k)\left[z_P^k(\delta_{it}^k - \delta_{rt}^k) + b_P^k(\xi_{it}^k - \xi_{rt}^k)\right]
\end{aligned}$$

Together, these expressions imply

$$cov_{ikt}(\bar{X}_{1i}^k, u_{it}^k) = \frac{1}{K}\sum_k \frac{o^k}{\varepsilon} z_P^k\, {z_s^k}^2\, var_i(\sigma^2_{\delta_i^k}) \tag{C.1}$$

$$cov_{ikt}(\bar{X}_{2i}^k, u_{it}^k) = \frac{1}{K}\sum_k \frac{o^k}{\varepsilon}\, {z_P^k}^2\, z_s^k var_i(\sigma^2_{\delta_i^k}) \tag{C.2}$$

which is the expression in the text, using once again assumption 5. Theorem 3 follows from assumption 6, since $\frac{1}{\varepsilon}\cdot\left(z_P^k\right)^2\cdot z_s^k = \frac{\varepsilon^k}{\varepsilon(1-\varepsilon^k\omega)^3} > 0$, $\frac{1}{\varepsilon}\cdot z_P^k\cdot\left(z_s^k\right)^2 = \frac{{\varepsilon^k}^2}{\varepsilon(1-\varepsilon^k\omega)^3} < 0$, $\frac{\partial\varepsilon}{\partial\psi_1} > 0$ and $\frac{\partial\varepsilon}{\partial\psi_2} < 0$.

THE CASE WITH HETEROGENEOUS SUPPLY ELASTICITY, $\omega^k = \omega + \rho^k$. — The previous result is now extended to the case where the elasticity of the supply curve is heterogenous across sectors, $\omega^k = \omega + \rho^k$. The residual of equation (13) can then be rewritten:

$$\begin{aligned}
u_{it}^k &= -\frac{1}{\varepsilon}(\xi_{it}^k - \xi_{rt}^k)(\delta_{it}^k - \delta_{rt}^k) + \frac{o^k}{\varepsilon}(\delta_{it}^k - \delta_{rt}^k)(d\ln P_{it}^k - d\ln P_{rt}^k) \\
&- \frac{\rho^k}{\varepsilon}(\xi_{it}^k - \xi_{rt}^k)(d\ln s_{it}^k - d\ln s_{rt}^k) + \frac{o^k\rho^k}{\varepsilon}(d\ln P_{it}^k - d\ln P_{rt}^k)(d\ln S_{it}^k - d\ln S_{rt}^k) \\
&= -\frac{1}{\varepsilon}(\xi_{it}^k - \xi_{rt}^k)(\delta_{it}^k - \delta_{rt}^k) + \frac{o^k}{\varepsilon}(\delta_{it}^k - \delta_{rt}^k)\left[z_P^k(\delta_{it}^k - \delta_{rt}^k) + b_P^k(\xi_{it}^k - \xi_{rt}^k)\right] \\
&- \frac{\rho^k}{\varepsilon}(\xi_{it}^k - \xi_{rt}^k)\left[z_s^k(\delta_{it}^k - \delta_{rt}^k) + b_s^k(\xi_{it}^k - \xi_{rt}^k)\right] \\
&+ \frac{o^k\rho^k}{\varepsilon}\left[z_P^k(\delta_{it}^k - \delta_{rt}^k) + b_P^k(\xi_{it}^k - \xi_{rt}^k)\right]\left[z_s^k(\delta_{it}^k - \delta_{rt}^k) + b_s^k(\xi_{it}^k - \xi_{rt}^k)\right]
\end{aligned}$$

Under assumption 5, the instruments for equation (13) are given by

$$\begin{aligned}
\bar{X}_{1i}^k &= \frac{1}{T}\sum_t \left[z_s^k(\delta_{it}^k - \delta_{rt}^k) + b_s^k(\xi_{it}^k - \xi_{rt}^k)\right]^2 = {z_s^k}^2\,\sigma_{\delta_i^k}^2 + {b_s^k}^2\,\sigma_{\xi_i^k}^2 \\
\bar{X}_{2i}^k &= \frac{1}{T}\sum_t \left[z_P^k(\delta_{it}^k - \delta_{rt}^k) + b_P^k(\xi_{it}^k - \xi_{rt}^k)\right]\left[z_s^k(\delta_{it}^k - \delta_{rt}^k) + b_s^k(\xi_{it}^k - \xi_{rt}^k)\right] \\
&= z_s^k z_P^k \sigma_{\delta_i^k}^2 + b_s^k b_P^k \sigma_{\xi_i^k}^2
\end{aligned}$$

Using the modified definition of u_{it}^k, these expressions can be used to obain

$$\begin{aligned}
cov_{ikt}(\bar{X}_{1i}^k, u_{it}^k) &= \frac{1}{K}\sum_k \frac{o^k}{\varepsilon} z_P^k\ {z_s^k}^2\,(1+\rho^k z_s^k) var_i(\sigma_{\delta_i^k}^2) \\
&- \frac{1}{K}\sum_k \frac{\rho^k}{\varepsilon}\ {b_s^k}^3\,(1 - o^k b_P^k) var_i(\sigma_{\xi_i^k}^2) \\
&+ \frac{1}{K}\sum_k \frac{o^k}{\varepsilon} z_P^k\ {b_s^k}^2\,(1+\rho^k z_s^k) cov_i(\sigma_{\delta_i^k}^2, \sigma_{\xi_i^k}^2) \\
&- \frac{1}{K}\sum_k \frac{\rho^k}{\varepsilon}\ {z_s^k}^2\, b_s^k (1 - o^k b_P^k) cov_i(\sigma_{\delta_i^k}^2, \sigma_{\xi_i^k}^2)
\end{aligned}$$

$$\begin{aligned}
cov_{ikt}(\bar{X}^k_{2i}, u^k_{it}) &= \frac{1}{K}\sum_k \frac{o^k}{\varepsilon}\ {z^k_P}^2\ z^k_s(1+\rho^k z^k_s) var_i(\sigma^2_{\delta^k_i}) \\
&- \frac{1}{K}\sum_k \frac{\rho^k}{\varepsilon}\ {b^k_s}^2\ b^k_P(1-o^k b^k_P) var_i(\sigma^2_{\xi^k_i}) \\
&+ \frac{1}{K}\sum_k \frac{o^k}{\varepsilon} z^k_P b^k_s b^k_P(1+\rho^k z^k_s) cov_i(\sigma^2_{\delta^k_i}, \sigma^2_{\xi^k_i}) \\
&- \frac{1}{K}\sum_k \frac{\rho^k}{\varepsilon} z^k_s z^k_P b^k_s(1-o^k b^k_P) cov_i(\sigma^2_{\delta^k_i}, \sigma^2_{\xi^k_i})
\end{aligned}$$

With assumption 5, the two expressions simplify into:

$$\begin{aligned}
cov_{ikt}(\bar{X}^k_{1i}, u^k_{it}) &= \frac{1}{K}\sum_k \frac{o^k}{\varepsilon} z^k_P\ {z^k_s}^2\ (1+\rho^k z^k_s) var_i(\sigma^2_{\delta^k_i}) \\
&\quad -\frac{1}{K}\sum_k \frac{\rho^k}{\varepsilon}\ {b^k_s}^3\ (1-o^k b^k_P) var_i(\sigma^2_{\xi^k_i}) \\
&= \frac{1}{K}\sum_k \frac{o^k}{\varepsilon} \frac{{\varepsilon^k}^2\ (1-\varepsilon^k\omega)}{(1-\varepsilon^k\omega^k)^4} var_i(\sigma^2_{\delta^k_i}) \\
&- \frac{1}{K}\sum_k \frac{\rho^k}{\varepsilon} \frac{1-\varepsilon^k\omega}{(1-\varepsilon^k\omega^k)^4} var_i(\sigma^2_{\xi^k_i})
\end{aligned} \tag{C.3}$$

$$\begin{aligned}
cov_{ikt}(\bar{X}^k_{2i}, u^k_{it}) &= \frac{1}{K}\sum_k \frac{o^k}{\varepsilon}\ {z^k_P}^2\ z^k_s(1+\rho^k z^k_s) var_i(\sigma^2_{\delta^k_i}) \\
&- \frac{1}{K}\sum_k \frac{\rho^k}{\varepsilon}\ {b^k_s}^2\ b^k_P(1-o^k b^k_P) var_i(\sigma^2_{\xi^k_i}) \\
&= \frac{1}{K}\sum_k \frac{o^k}{\varepsilon} \frac{\varepsilon^k(1-\varepsilon^k\omega)}{(1-\varepsilon^k\omega^k)^4} var_i(\sigma^2_{\delta^k_i}) \\
&- \frac{1}{K}\sum_k \frac{\rho^k}{\varepsilon} \frac{\omega^k(1-\varepsilon\omega^k)}{(1-\varepsilon^k\omega^k)^4} var_i(\sigma^2_{\xi^k_i})
\end{aligned} \tag{C.4}$$

By definition, $\frac{1}{\varepsilon}\frac{{\varepsilon^k}^2(1-\varepsilon^k\omega)}{(1-\varepsilon^k\omega^k)^4} < 0$, and $\frac{1}{\varepsilon}\frac{\varepsilon^k(1-\varepsilon^k\omega)}{(1-\varepsilon^k\omega^k)^4} > 0$. Thus, the first terms in both $cov_{ikt}(\bar{X}^k_{1i}, u^k_{it})$ and $cov_{ikt}(\bar{X}^k_{2i}, u^k_{it})$ have the same properties as in the previous case where ω was homogeneous across sectors: Under assumption 6, the estimated price elasticity of trade is systematically biased towards zero in pooled sectoral data. The second terms in both expressions pertain to the heterogeneity in ω^k. By definition, both $\frac{1}{\varepsilon}\frac{1-\varepsilon^k\omega}{(1-\varepsilon^k\omega^k)^4}$, and $\frac{1}{\varepsilon}\frac{\omega^k(1-\varepsilon\omega^k)}{(1-\varepsilon^k\omega^k)^4}$ are negative. Therefore, the bias that arises from the heterogeneity in ω^k affects ψ_1 and ψ_2 in the same direction, with ambiguous end effects.

C2. Aggregate estimate

THE CASE WITH HOMOGENEOUS SUPPLY ELASTICITY $\omega^k = \omega$. — Using assumptions 6-7-8 in equations (11)-(12), it is easy to compute the instruments for equation (14):

$$\begin{aligned}
\bar{X}_{1i} &= \frac{1}{T}\sum_t \left[\sum_k m^k[z_s^k(\delta_{it}^k - \delta_{rt}^k) + b_s^k(\xi_{it}^k - \xi_{rt}^k)]\right]^2 \\
&= \sum_k \sum_{k'} m^k m^{k'} \left[z_s^k z_s^{k'} \sigma_{\delta_i^k \delta_i^{k'}} + b_s^k b_s^{k'} \sigma_{\xi_i^k \xi_i^{k'}}\right] \\
\bar{X}_{2i} &= \frac{1}{T}\sum_t \left[\sum_k m^k[z_P^k(\delta_{it}^k - \delta_{rt}^k) + b_P^k(\xi_{it}^k - \xi_{rt}^k)]\right]\left[\sum_k m^k[z_s^k(\delta_{it}^k - \delta_{rt}^k) + b_s^k(\xi_{it}^k - \xi_{rt}^k)]\right] \\
&= \sum_k \sum_{k'} m^k m^{k'} \left[z_s^k z_P^{k'} \sigma_{\delta_i^k \delta_i^{k'}} + b_s^k b_P^{k'} \sigma_{\xi_i^k \xi_i^{k'}}\right]
\end{aligned}$$

where $\sigma_{\delta_i^k \delta_i^{k'}} \equiv cov_t(\delta_{it}^k - \delta_{rt}^k, \delta_{it}^{k'} - \delta_{rt}^{k'})$, and $\sigma_{\xi_i^k \xi_i^{k'}} \equiv cov_t(\xi_{it}^k - \xi_{rt}^k, \xi_{it}^{k'} - \xi_{rt}^{k'})$. This uses assumption 7, which implies $cov_t(\delta_{it}^k - \delta_{rt}^k, \xi_{it}^{k'} - \xi_{rt}^{k'}) = 0,\ \forall k, k'$.

The residual of the aggregate equation writes:

$$\begin{aligned}
u_{it} &= -\frac{1}{\varepsilon}\left(\sum_k m^k(\xi_{it}^k - \xi_{rt}^k)\right)\left(\sum_k m^k(\delta_{it}^k - \delta_{rt}^k)\right) \\
&+ \frac{1}{\varepsilon}\left(\sum_k m^k(\delta_{it}^k - \delta_{rt}^k)\right)\left(\sum_k m^k o^k(d\ln P_{it}^k - d\ln P_{rt}^k)\right) \\
&= -\frac{1}{\varepsilon}\left(\sum_k m^k(\xi_{it}^k - \xi_{rt}^k)\right)\left(\sum_k m^k(\delta_{it}^k - \delta_{rt}^k)\right) \\
&+ \frac{1}{\varepsilon}\left(\sum_k m^k(\delta_{it}^k - \delta_{rt}^k)\right)\left(\sum_k m^k o^k[z_P^k(\delta_{it}^k - \delta_{rt}^k) + b_P^k(\xi_{it}^k - \xi_{rt}^k)]\right)
\end{aligned}$$

Together, these expressions imply

$$cov_{it}(\bar{X}_{1i}, u_{it}) = \frac{1}{\varepsilon} \sum_k \sum_{k'} \sum_{k''} \sum_{k'''} m^k m^{k'} m^{k''} m^{k'''} o^k z_P^k z_s^{k''} z_s^{k'''}$$
$$\times cov_i(\sigma_{\delta_i^k \delta_i^{k'}}, \sigma_{\delta_i^{k''} \delta_i^{k'''}}) + \frac{1}{\varepsilon} \sum_k \sum_{k'} \sum_{k''} \sum_{k'''} m^k m^{k'} m^{k''} m^{k'''}$$
$$\times o^k z_P^k b_s^{k''} b_s^{k'''} cov_i(\sigma_{\delta_i^k \delta_i^{k'}}, \sigma_{\xi_i^{k''} \xi_i^{k'''}})$$
$$cov_{it}(\bar{X}_{2i}, u_{it}) = \frac{1}{\varepsilon} \sum_k \sum_{k'} \sum_{k''} \sum_{k'''} m^k m^{k'} m^{k''} m^{k'''} o^k z_P^k z_s^{k''} z_P^{k'''}$$
$$\times cov_i(\sigma_{\delta_i^k \delta_i^{k'}}, \sigma_{\delta_i^{k''} \delta_i^{k'''}}) + \frac{1}{\varepsilon} \sum_k \sum_{k'} \sum_{k''} \sum_{k'''} m^k m^{k'} m^{k''} m^{k'''}$$
$$\times o^k z_P^k b_s^{k''} b_P^{k'''} cov_i(\sigma_{\delta_i^k \delta_i^{k'}}, \sigma_{\xi_i^{k''} \xi_i^{k'''}})$$

With assumption 7, the expressions simplify into

$$\begin{aligned} cov_{it}(\bar{X}_{1i}, u_{it}) &= \sum_k \left(m^k\right)^4 \cdot \frac{o^k}{\varepsilon} \cdot z_P^k \cdot \left(z_s^k\right)^2 \cdot var_i(\sigma^2_{\delta_i^k}) \\ cov_{it}(\bar{X}_{2i}, u_{it}) &= \sum_k \left(m^k\right)^4 \cdot \frac{o^k}{\varepsilon} \cdot \left(z_P^k\right)^2 \cdot z_s^k \cdot var_i(\sigma^2_{\delta_i^k}) \end{aligned}$$

where $\sigma^2_{\delta_i^k} = \sigma_{\delta_i^k \delta_i^k}$ and $\sigma^2_{\xi_i^k} = \sigma_{\xi_i^k \xi_i^k}$ denote the variances of supply and demand shocks, computed over time within sector k in country i. This is the expression in the text. Theorem 4 follows from assumption 6, since $\frac{1}{\varepsilon} \cdot \left(z_P^k\right)^2 \cdot z_s^k = \frac{\varepsilon^k}{\varepsilon(1-\varepsilon^k\omega)^3} > 0$, and $\frac{1}{\varepsilon} \cdot z_P^k \cdot \left(z_s^k\right)^2 = \frac{\varepsilon^{k\,2}}{\varepsilon(1-\varepsilon^k\omega)^3} < 0$.

The case with heterogeneous supply elasticity, $\omega^k = \omega + \rho^k$. — The previous results are now extended to the case in which the elasticity of the supply curve is heterogenous across sectors, $\omega^k = \omega + \rho^k$. The residual of equation (14) can then be rewritten

$$
\begin{aligned}
u_{it} &= -\frac{1}{\varepsilon}\left(\sum_k m^k(\xi_{it}^k-\xi_{rt}^k)\right)\left(\sum_k m^k(\delta_{it}^k-\delta_{rt}^k)\right) \\
&+ \frac{1}{\varepsilon}\left(\sum_k m^k(\delta_{it}^k-\delta_{rt}^k)\right)\left(\sum_k m^k o^k(d\ln P_{it}^k-d\ln P_{rt}^k)\right) \\
&- \frac{1}{\varepsilon}\left(\sum_k m^k(\xi_{it}^k-\xi_{rt}^k)\right)\left(\sum_k m^k \rho^k(d\ln s_{it}^k-d\ln s_{rt}^k)\right) \\
&+ \frac{1}{\varepsilon}\left(\sum_k m^k o^k(d\ln P_{it}^k-d\ln P_{rt}^k)\right)\left(\sum_k m^k \rho^k(d\ln S_{it}^k-d\ln S_{rt}^k)\right)
\end{aligned}
$$

while under assumptions 6-7-8, the instruments for equation (14) are given by

$$
\begin{aligned}
\bar{X}_{1i} &= \frac{1}{T}\sum_t\left[\sum_k m^k[z_s^k(\delta_{it}^k-\delta_{rt}^k)+b_s^k(\xi_{it}^k-\xi_{rt}^k)]\right]^2 \\
&= \sum_k\sum_{k'} m^k m^{k'}\left[z_s^k z_s^{k'}\sigma_{\delta_i^k\delta_i^{k'}}+b_s^k b_s^{k'}\sigma_{\xi_i^k\xi_i^{k'}}\right] \\
\bar{X}_{2i} &= \frac{1}{T}\sum_t\left[\sum_k m^k[z_P^k(\delta_{it}^k-\delta_{rt}^k)+b_P^k(\xi_{it}^k-\xi_{rt}^k)]\right]\left[\sum_k m^k[z_s^k(\delta_{it}^k-\delta_{rt}^k)+b_s^k(\xi_{it}^k-\xi_{rt}^k)]\right] \\
&= \sum_k\sum_{k'} m^k m^{k'}\left[z_s^k z_P^{k'}\sigma_{\delta_i^k\delta_i^{k'}}+b_s^k b_P^{k'}\sigma_{\xi_i^k\xi_i^{k'}}\right]
\end{aligned}
$$

Using the modified definition of u_{it}, the three expressions can be used to obain

$$
\begin{aligned}
&cov_{it}(\bar{X}_{1i},u_{it}) = \frac{1}{\varepsilon}\sum_k\sum_{k'}\sum_{k''}\sum_{k'''} m^k m^{k'} m^{k''} m^{k'''} o^k z_P^k(1+\rho^{k'}z_s^{k'})z_s^{k''}z_s^{k'''} \\
&\times cov_i(\sigma_{\delta_i^k\delta_i^{k'}},\sigma_{\delta_i^{k''}\delta_i^{k'''}}) - \frac{1}{\varepsilon}\sum_k\sum_{k'}\sum_{k''}\sum_{k'''} m^k m^{k'} m^{k''} m^{k'''} \rho^k b_s^k b_s^{k''} b_s^{k'''} \\
&\times(1-o^{k'}b_P^{k'})cov_i(\sigma_{\xi_i^k\xi_i^{k'}},\sigma_{\xi_i^{k''}\xi_i^{k'''}}) + \frac{1}{\varepsilon}\sum_k\sum_{k'}\sum_{k''}\sum_{k'''} m^k m^{k'} m^{k''} m^{k'''} \\
&\times o^k z_P^k(1+\rho^{k'}z_s^{k'})b_s^{k''}b_s^{k'''}cov_i(\sigma_{\delta_i^k\delta_i^{k'}},\sigma_{\xi_i^{k''}\xi_i^{k'''}}) - \frac{1}{\varepsilon}\sum_k\sum_{k'}\sum_{k''}\sum_{k'''} m^k \\
&\times m^{k'} m^{k''} m^{k'''} z_s^k z_s^{k'} \rho^{k''} b_s^{k''}(1-o^{k'''}b_P^{k'''})cov_i(\sigma_{\delta_i^k\delta_i^{k'}},\sigma_{\xi_i^{k''}\xi_i^{k'''}})
\end{aligned}
$$

$$cov_{it}(\bar{X}_{2i}, u_{it}) = \frac{1}{\varepsilon}\sum_k \sum_{k'} \sum_{k''} \sum_{k'''} m^k m^{k'} m^{k''} m^{k'''} o^k z_P^k (1+\rho^{k'} z_s^{k'}) z_s^{k''} z_P^{k'''}$$
$$\times cov_i(\sigma_{\delta_i^k \delta_i^{k'}}, \sigma_{\delta_i^{k''} \delta_i^{k'''}}) - \frac{1}{\varepsilon}\sum_k \sum_{k'} \sum_{k''} \sum_{k'''} m^k m^{k'} m^{k''} m^{k'''} \rho^k b_s^k b_s^{k''} b_P^{k'''}$$
$$\times (1 - o^{k'} b_P^{k'}) cov_i(\sigma_{\xi_i^k \xi_i^{k'}}, \sigma_{\xi_i^{k''} \xi_i^{k'''}}) + \frac{1}{\varepsilon}\sum_k \sum_{k'} \sum_{k''} \sum_{k'''} m^k m^{k'} m^{k''} m^{k'''}$$
$$\times o^k z_P^k (1+\rho^{k'} z_s^{k'}) b_s^{k''} b_P^{k'''} cov_i(\sigma_{\delta_i^k \delta_i^{k'}}, \sigma_{\xi_i^{k''} \xi_i^{k'''}}) - \frac{1}{\varepsilon}\sum_k \sum_{k'} \sum_{k''} \sum_{k'''} m^k m^{k'}$$
$$\times m^{k''} m^{k'''} z_s^k z_P^{k'} \rho^{k''} b_s^{k''} (1 - o^{k'''} b_P^{k'''}) cov_i(\sigma_{\delta_i^k \delta_i^{k'}}, \sigma_{\xi_i^{k''} \xi_i^{k'''}})$$

Under assumption 7, the two expressions simplify into:

$$\begin{aligned}
cov_{it}(\bar{X}_{1i}, u_{it}) &= \sum_k {m^k}^4 \frac{o^k}{\varepsilon} z_P^k {z_s^k}^2 (1+\rho^k z_s^k) var_i(\sigma^2_{\delta_i^k}) \\
&\quad - \sum_k {m^k}^4 \frac{\rho^k}{\varepsilon} {b_s^k}^3 (1 - o^k b_P^k) var_i(\sigma^2_{\xi_i^k}) \\
cov_{it}(\bar{X}_{2i}, u_{it}) &= \sum_k {m^k}^4 \frac{o^k}{\varepsilon} {z_P^k}^2 z_s^k (1+\rho^k z_s^k) var_i(\sigma^2_{\delta_i^k}) \\
&\quad - \sum_k {m^k}^4 \frac{\rho^k}{\varepsilon} {b_s^k}^2 b_P^k (1 - o^k b_P^k) var_i(\sigma^2_{\xi_i^k})
\end{aligned}$$

Finally, use the expressions for z_P^k, z_s^k, b_P^k, and b_s^k to obtain

$$\begin{aligned}
cov_{it}(\bar{X}_{1i}, u_{it}) &= \sum_k {m^k}^4 \frac{o^k}{\varepsilon} \frac{{\varepsilon^k}^2 (1-\varepsilon^k \omega)}{(1-\varepsilon^k \omega^k)^4} var_i(\sigma^2_{\delta_i^k}) \\
&\quad - \sum_k {m^k}^4 \frac{\rho^k}{\varepsilon} \frac{1-\varepsilon^k \omega}{(1-\varepsilon^k \omega^k)^4} var_i(\sigma^2_{\xi_i^k}) \\
cov_{it}(\bar{X}_{2i}, u_{it}) &= \sum_k {m^k}^4 \frac{o^k}{\varepsilon} \frac{\varepsilon^k (1-\varepsilon^k \omega)}{(1-\varepsilon^k \omega^k)^4} var_i(\sigma^2_{\delta_i^k}) \\
&\quad - \sum_k {m^k}^4 \frac{\rho^k}{\varepsilon} \frac{\omega^k (1-\varepsilon \omega^k)}{(1-\varepsilon^k \omega^k)^4} var_i(\sigma^2_{\xi_i^k})
\end{aligned}$$

By definition, $\frac{1}{\varepsilon}\frac{{\varepsilon^k}^2(1-\varepsilon^k\omega)}{(1-\varepsilon^k\omega^k)^4} < 0$, and $\frac{1}{\varepsilon}\frac{\varepsilon^k(1-\varepsilon^k\omega)}{(1-\varepsilon^k\omega^k)^4} > 0$. Thus, the first terms in both $cov_{it}(\bar{X}_{1i}, u_{it})$ and $cov_{it}(\bar{X}_{2i}, u_{it})$ have the same properties as in the previous section: under assumption 6, the estimated price elasticity of trade is systematically biased towards zero in aggregate data. The second terms in both expressions pertain to the heterogeneity in ω^k. By definition, both $\frac{1}{\varepsilon}\frac{1-\varepsilon^k\omega}{(1-\varepsilon^k\omega^k)^4}$, and $\frac{1}{\varepsilon}\frac{\omega^k(1-\varepsilon\omega^k)}{(1-\varepsilon^k\omega^k)^4}$ are negative. Therefore, the bias that arises from the heterogeneity in ω^k affects ψ_1 and ψ_2 in the same direction, with ambiguous end effects on $\hat{\varepsilon}$.

Approximating Aggregate Market Shares

Consider the definition of the market share of country i in aggregate imports:

$$s_{it} = \frac{\sum_{k=1}^K P_{it}^k C_{it}^k}{\sum_{i=1,i\neq j}^I \sum_{k=1}^K P_{it}^k C_{it}^k} = \sum_{k=1}^K \frac{P_{it}^k C_{it}^k}{\sum_{i=1,i\neq j}^I P_{it}^k C_{it}^k} m_t^k = \sum_{k=1}^K m_t^k s_{it}^k$$

where $m_t^k = \frac{\sum_{i=1,i\neq j}^I P_{it}^k C_{it}^k}{\sum_{i=1,i\neq j}^I \sum_{k=1}^K P_{it}^k C_{it}^k}$ is the expenditure share of sector k in aggregate imports, and j denotes the domestic economy. The growth rate of the aggregate market share of country i reflects the changes in the market share of country i in each sector s_{it}^k, and the changes in sector shares. For small changes:

$$\begin{aligned}
d\ln s_{it} &= \ln\sum_{k=1}^K m_t^k s_{it}^k - \ln\sum_{k=1}^K m_{t-1}^k s_{it-1}^k \simeq \frac{\sum_{k=1}^K m_t^k s_{it}^k}{\sum_{k=1}^K m_{t-1}^k s_{it-1}^k} - 1 \\
&= \sum_k m_t^k \left(\frac{s_{it}^k}{s_{it-1}^k} - 1\right)\frac{s_{it-1}^k}{\sum_k m_{t-1}^k s_{it-1}^k} + \sum_k m_t^k \frac{s_{it-1}^k}{\sum_k m_{t-1}^k s_{it-1}^k} - 1 \\
&\simeq \sum_k d\ln s_{it}^k \frac{\sum_{i\neq j} P_{it}^k C_{it}^k}{\sum_{i\neq j}\sum_k P_{it}^k C_{it}^k}\frac{P_{it-1}^k C_{it-1}^k}{\sum_{i\neq j} P_{it-1}^k C_{it-1}^k}\left[\sum_k \frac{\sum_{i\neq j} P_{it-1}^k C_{it-1}^k}{\sum_{i\neq j}\sum_k P_{it-1}^k C_{it-1}^k}\frac{P_{it-1}^k C_{it-1}^k}{\sum_{i\neq j} P_{it-1}^k C_{it-1}^k}\right]^{-1} \\
&+ \sum_k d\ln m_t^k \frac{m_{t-1}^k s_{it-1}^k}{\sum_k m_{t-1}^k s_{it-1}^k} \\
&= \sum_k m_{it-1}^k d\ln s_{it}^k + \sum_{p,k} m_{it-1}^k d\ln m_t^k
\end{aligned}$$

where $m_{it}^k = \frac{m_t^k s_{it}^k}{\sum_{k=1}^K m_t^k s_{it}^k} = \frac{P_{it}^k C_{it}^k}{\sum_{k=1}^K P_{it}^k C_{it}^k}$. If sectoral expenditures m_{it}^k are constant, this implies:

$$d\ln s_{it} = \sum_k m_i^k d\ln s_{it}^k$$

A Four-Sector Version of BKK[37]

Each country $j = 1, 2$ is inhabited by a large number of identical agents and labor is internationally immobile. The main departure from BKK is that each country produces four goods, $k = 1, 2, 3, 4$. Preferences of the representative agent in country j are characterized by utility of the form

$$E_0 \sum_{t=0}^{\infty} b^t U(C_{jt}, 1 - N_{jt})$$

where b is a discount factor, $U = \log C + \xi \frac{(1-N)^{1-\varkappa}}{1-\varkappa}$ and C_{jt} (N_{jt}) denote aggregate consumption (hours worked). Aggregate consumption is a Cobb-Douglas function of sector-specific consumptions. The same structure prevails for aggregate investment:

$$C_{jt} = \prod_{k=1}^{4} \left(\frac{C_{jt}^k}{\alpha_j^k} \right)^{\alpha_j^k}, \qquad I_{jt} = \prod_{k=1}^{4} \left(\frac{I_{jt}^k}{\alpha_j^k} \right)^{\alpha_j^k}$$

where C_{jt}^k is the consumption basket of good k and α_j^k its share in nominal consumption. I_{jt}^k is investment in sector k.

Sectoral output is produced with capital K and labor N following a Cobb-Douglas function:

$$Y_{jt}^k = Z_{jt} \left(K_{jt}^k \right)^{\kappa} \left(N_{jt}^k \right)^{1-\kappa}, \quad j = 1, 2, \quad k = 1, 2, 3, 4$$

The quantity Y_{jt}^k denotes country j's production of good k, in units of the local good. In equilibrium, it is equal to domestic sales $C_{jjt}^k + I_{jjt}^k$ plus exports $C_{jit}^k + I_{jit}^k$. The matrix $\mathbf{Z}_t = (Z_{1t}, Z_{2t})$ denotes country-specific shocks to productivity. Productivity shocks are aggregate, so that producer prices are homogenous across sectors. In what follows, domestic prices are normalized to unity and the relative price of foreign goods is denoted P.

Sectoral consumption and investment, C_{jt}^k and I_{jt}^k are composites of foreign and domestic goods:

$$C_{jt}^k = \left[\sum_{i=1}^{2} \left(\beta_{ij}^k C_{ijt}^k \right)^{\frac{\varepsilon^k}{\varepsilon^k - 1}} \right]^{\frac{\varepsilon^k - 1}{\varepsilon^k}}, \qquad I_{jt}^k = \left[\sum_{i=1}^{2} \left(\beta_{ij}^k I_{ijt}^k \right)^{\frac{\varepsilon^k}{\varepsilon^k - 1}} \right]^{\frac{\varepsilon^k - 1}{\varepsilon^k}}$$

The elasticity of substitution between foreign and domestic varieties is sector-specific. The weights β_{ij}^k are related to the share of imports in the sectoral consumption of good k. In the calibration, they are assumed symmetric across coun-

[37]We are grateful to Jean-Olivier Hairault for sharing his codes to solve the one-sector version of BKK.

tries but can differ across sectors.

The aggregate capital stock evolves in each country according to: $K_{jt+1} = (1-a)K_{jt} + I_{jt}$ where a is the depreciation rate. Adjustment costs for capital are given by:

$$\mathcal{C}_{jt} = \frac{\Phi}{2}\frac{(K_{jt+1} - K_{jt})^2}{K_{jt}}$$

Finally, fluctuations arise from persistent shocks to aggregate productivity: $\mathbf{Z}_{t+1} = \mathbf{A}\mathbf{Z}_t + \epsilon^Z_{t+1}$ where ϵ^Z is distributed normally and independently over time with variance $\mathbf{V}^Z$. The correlation between the technology shocks, $\mathbf{Z}_1$ and $\mathbf{Z}_2$ is determined by the off-diagonal elements of $\mathbf{A}$ and $\mathbf{V}^Z$.

Aggregate GDP in country j in period t, in units of domestically produced goods, is $Y_{jt} = \sum_{k=1}^{4} Y^k_{jt}$. The resource constraint equates sectoral GDPs to the sum of (domestic and foreign) consumption and investment:

$$Y^k_{jt} = C^k_{jjt} + C^k_{jit} + I^k_{jjt} + I^k_{jit}, \quad k = 1, 2, 3, 4$$

National output is related to the expenditure components according to:

$$Y_{jt} = \sum_{k=1}^{4}(C^k_{jjt} + I^k_{jjt}) + P_t\left[\sum_{k=1}^{4}(C^k_{ijt} + I^k_{ijt})\right]$$

Finally, the trade balance, defined as the ratio of net exports to output, both measured in current prices, is:

$$nx_t = \frac{\sum_{k=1}^{4}(C^k_{jit} + I^k_{jit}) - P_t\left[\sum_{k=1}^{4}(C^k_{ijt} + I^k_{ijt})\right]}{Y_{jt}}$$

and the terms of trade P_t equal the sectoral marginal rate of transformation between the two varieties in country 1, evaluated at equilibrium quantities.

Table E.1 summarizes the calibration of the key parameters. Elasticities of substitution are defined at the sector level, which is the main deviation from BKK. In a symmetric steady state with $P = 1$, the values of β^k_{ij} are linked with λ^k_j according to

$$\beta^k_{ij} = \left[\left(\frac{\lambda^k_j}{1-\lambda^k_j}\right)^{\frac{1}{1-\rho_k}} + 1\right]^{-1} \quad \textit{and} \quad \beta^k_{jj} = 1 - \beta^k_{ij}$$

Table E.1—Benchmark Parameter Values

Preferences	
[a]Discount rate	$b = 0.99$
[a]Labor supply elasticity	$\varkappa = 5$
[b]Sectoral elasticities of substitution	$\{1 - \varepsilon^k\} = (2.9, 3.6, 4.5, 10.5)$
[b]Share of each sector in consumption	$\{\alpha_j^k\} = (.280, .283, .257, .180)$
[b]Sectoral import shares	$\{\lambda_j^k\} = (.180, .204, .283, .252)$
Technology	
[a]Share of capital in total costs	$\kappa = 0.36$
[a]Depreciation rate	$a = 0.025$
[a]Adjustment cost	$\Phi = 10^{-6}$
[a]SS hours worked	$N_{jSS} = 0.34$
[a]SS hours worked in k	$N_{jSS}^k = \alpha_j^k N_{jSS}$
Forcing processes	
[a]Correlation matrix	$\mathbf{A} = \begin{bmatrix} 0.906 & 0.088 \\ 0.088 & 0.906 \end{bmatrix}$
[a]Variance of productivity shocks	$Var(\epsilon_1^Z) = Var(\epsilon_2^Z) = 0.00852^2$
[a]Cross-country correlation of productivity shocks	$Corr(\epsilon_1^Z, \epsilon_2^Z) = 0.258$

Note: [a] indicates a parameter value taken from BKK. [b] indicates a parameter value calibrated from US data. Import shares λ_j^k are used to calibrate the weight parameters of the sectoral consumption functions (β_{ij}^k) using $P = \left(\frac{1-\lambda_j^k}{\lambda_j^k}\right)^{1/\rho^k} \left(\frac{\beta_{ij}^k}{\beta_{jj}^k}\right)^{\frac{\rho^k-1}{\rho^k}}$, $\beta_{ij}^k + \beta_{jj}^k = 1$ and $P = 1$ in the steady state.